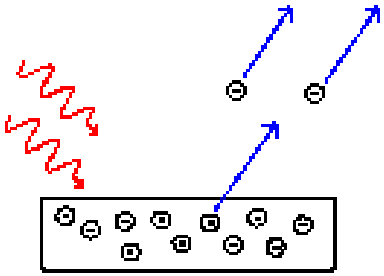
Particle-wave Dualism

1905 Photoelectric Effect



$$E = f\hbar$$

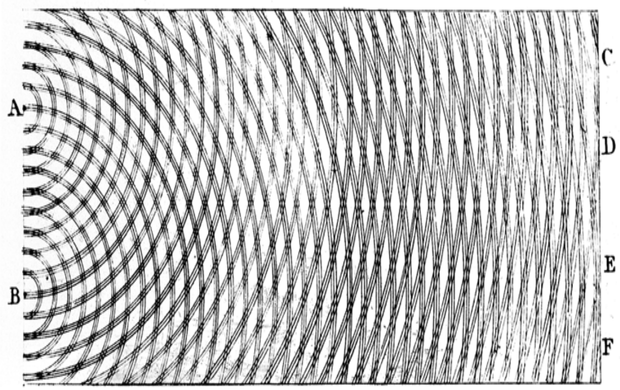
$$\hbar = 6.626x10^{-34}\,J$$

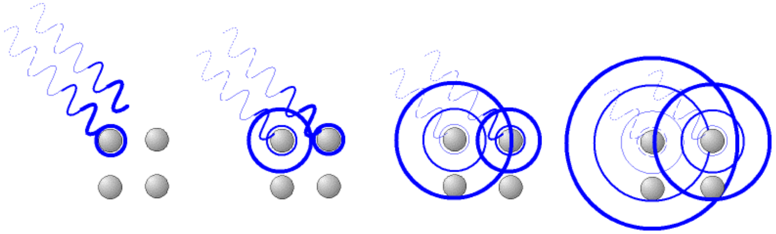
$$f = frequency$$

$$\lambda = \frac{h}{p}$$

De Broglie Hypothesis

Wave Interference





$$n\lambda = 2d \cdot \sin\theta$$

Uncertainty Principle

$$(\Delta x)(\Delta p_x) \geq \hbar/2$$

$$(\Delta y)(\Delta p_y) \geq \hbar/2$$

$$(\Delta z)(\Delta p_z) \geq \hbar/2$$

Schrödinger Equation

$$E = \hbar\omega$$

$$\mathbf{p} = \hbar\mathbf{k}$$

$$\psi \approx e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

$$\frac{\partial}{\partial x}\psi = ik_x\psi$$

$$\frac{\partial}{\partial t}\psi = -i\omega\psi$$

$$E\psi = \hbar\omega\psi = i\hbar\frac{\partial}{\partial t}\psi$$

$$p_x\psi = \hbar k_x\psi = -i\hbar\frac{\partial}{\partial x}\psi$$

$$p_x^2\psi = -\hbar^2\frac{\partial^2}{\partial x^2}\psi$$

$$p^2\psi = (p_x^2 + p_y^2 + p_z^2)\psi = -\hbar^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = -\hbar^2\nabla^2\psi$$

$$E = \frac{p^2}{2m} + V$$

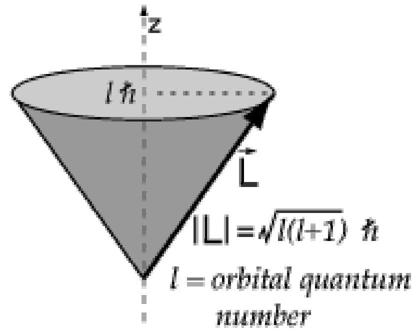
$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

Quantum Theory of the Angular Momentum

$$l = r \ x \ p \qquad p_x = -i\hbar\partial / \partial x, \ p_y = -i\hbar\partial / \partial y..$$

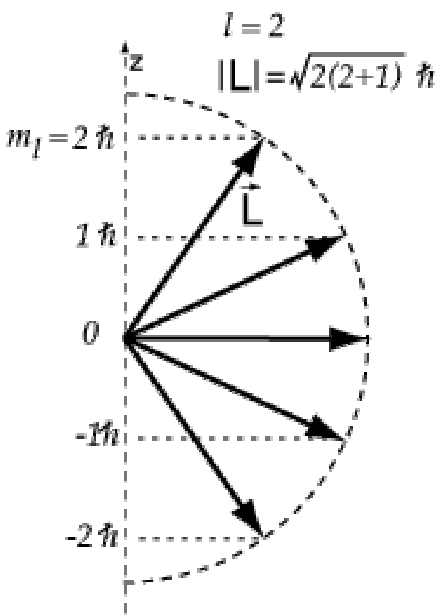
$$l_x = yp_z - zp_y$$

$$\langle l^2 \rangle = \langle l_x^2 + l_y^2 + l_z^2 \rangle = \hbar^2 l(l+1)$$



The angular Momentum is a constant of the motion

Only 1 component can be determined at the same time→ Precession



$$\langle l_z \rangle = \hbar m_l$$

$$m_l = 0, \pm 1, \pm 2 \ldots \pm l$$

Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	-1.9130418
p	$+2.7928456$
^{2}H (D)	$+0.8574376$
^{17}O	-1.89379
^{57}Fe	$+0.09062293$
^{57}Co	$+4.733$
^{93}Nb	$+6.1705$

All values refer to the nuclear ground states; uncertainties are typically a few parts in the last digit. For a complete tabulation, see V. S. Shirley, in *Table of Isotopes* (Wiley: New York, 1978), Appendix VII.

Hamilton operator without hyperfine interaction,
without external magnetic field

$$H = H_0$$

With hyperfine interaction:

$$H = H_0 + A\vec{I} \cdot \vec{J} \;, \qquad A = -\frac{\mu_I B_J(0)}{IJ}$$

$$\vec{F} = \vec{I} + \vec{J}, \qquad \left[H, F^2\right] = 0, \qquad \left[H, F_z\right] = 0$$

The eigenvalues of $\vec{I} \cdot \vec{J}$ are $\frac{1}{2}\left[F(F+1) - I(I+1) - J(J+1)\right]$

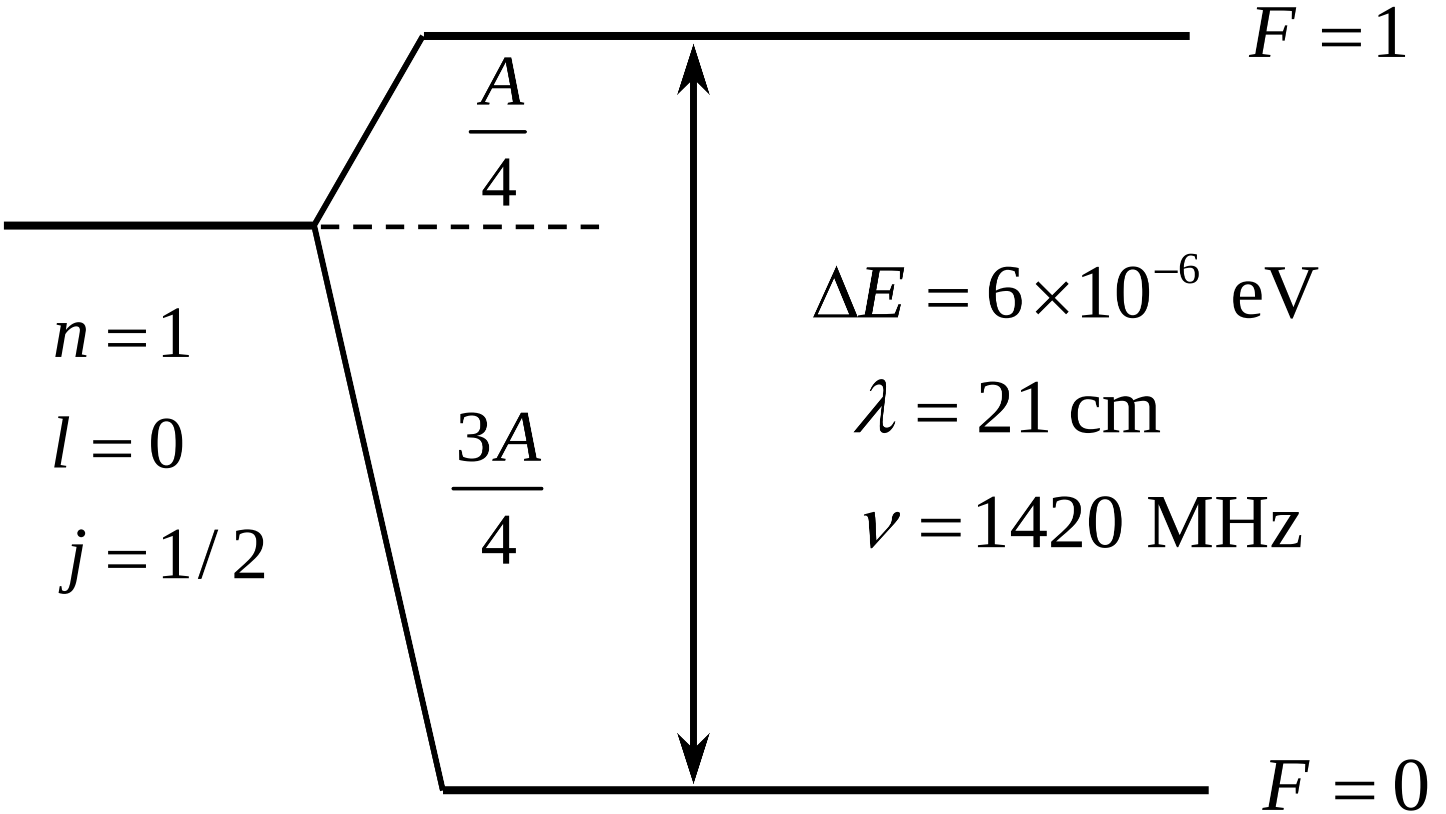
$\Rightarrow$ energetic splitting of $F-$ states

$\Rightarrow$ Hyperfine structure of the atomic spectra

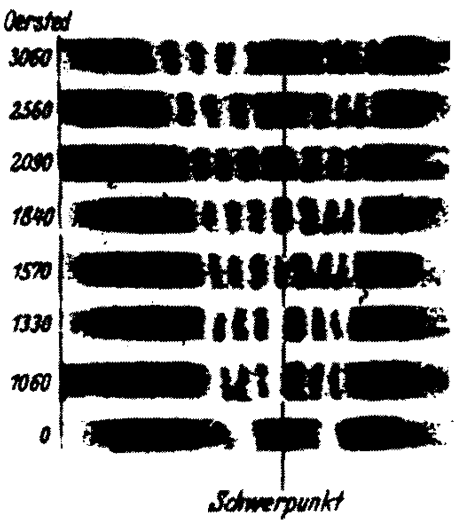
Magnetic field at the nucleus (in Tesla), produced by atomic electron:

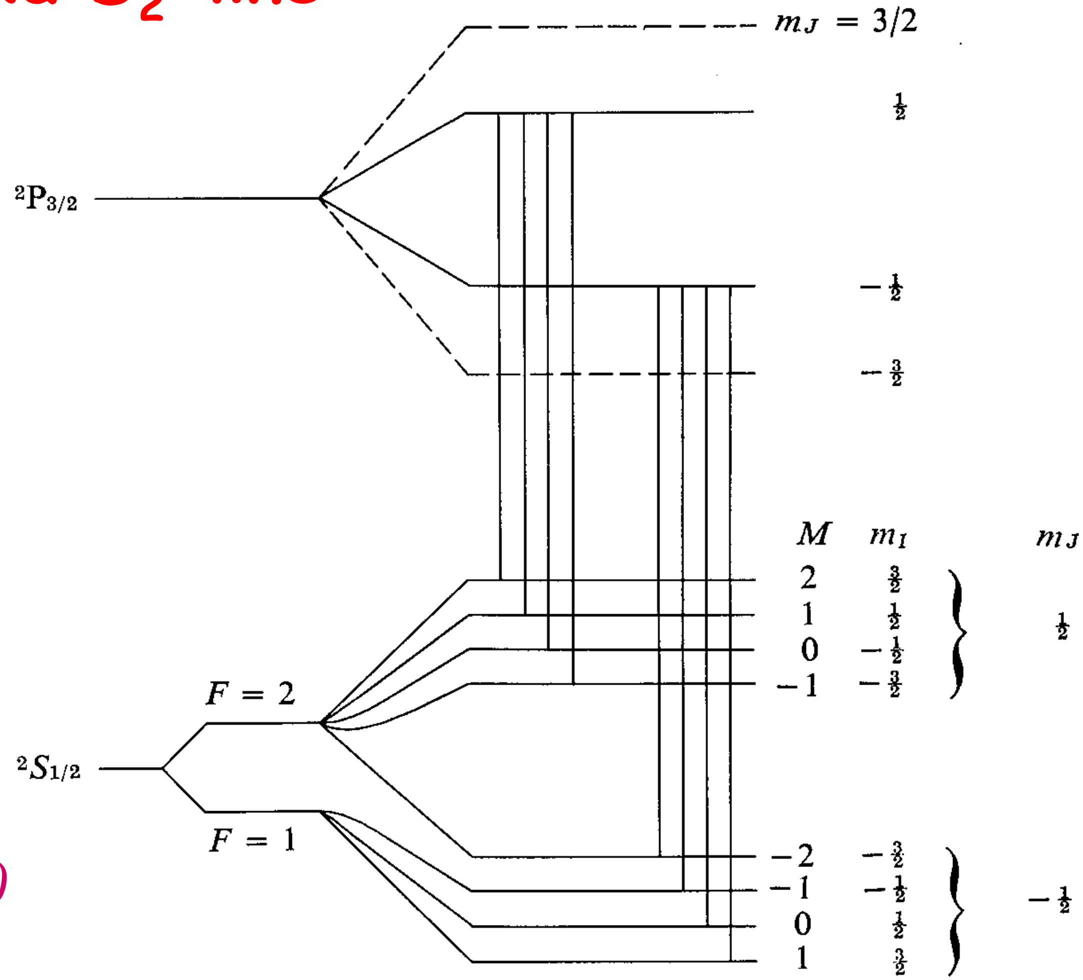
Atom	n	$^2S_{1/2}$	$^2P_{1/2}$	$^2P_{3/2}$
H	1	17.4		
Na	3	44	4.2	2.5
K	4	63	7.9	4.6
Cs	6	210	28	13

Hyperfine splitting of the hydrogen ground state



Effect of a magnetic field on the hyperfine-splitting of the Na D_2-line





(Segrè, Nuclei & particles)

Figure 6-27 The line $^2S_{1/2} - {}^2P_{3/2}$ (5890) of Na as observed in absorption. The nuclear spin $I = \frac{3}{2}$ splits the terms as indicated in the right side of the figure. Note that in the $^2P_{3/2}$ term the hfs is so small that the strong external field case applies always. The observations have been made for various values of an external field H as indicated in the right part of the figure. [D. A. Jackson and H. Kuhn, *Proc. Roy. Soc.* (*London*), **167,** 210 (1938).]

With magnetic field $\vec{H}_e$: $H = H_0 + A\vec{I}\cdot\vec{J} - \vec{\mu}_J\cdot\vec{H}_e - \vec{\mu}_I\cdot\vec{H}_e$

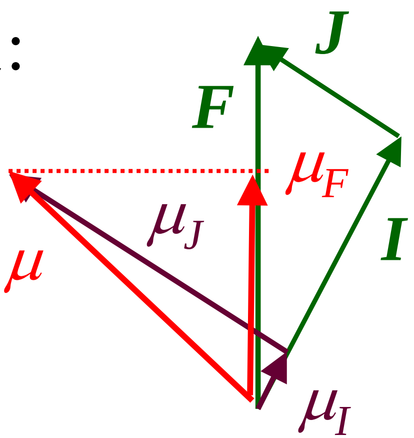
$$= H_0 + H_{HF} + H_{Zeeman}$$

(a) weak field: $|H_{Zeeman}| \,\square\, |H_{HF}| \rightarrow \left[H,\vec{F}\right] \approx 0$

H_{Zeeman} lifts degeneracy of states with different F_z:

$$\Delta W_{Zeeman} = g_F \mu_B H_e m_F$$

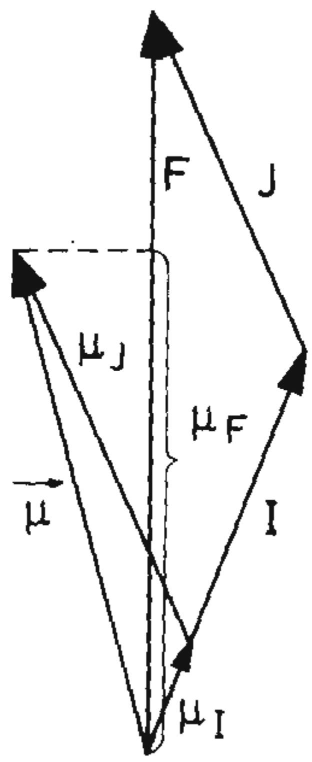
$$g_F = g_J \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} - g_I \frac{\mu_N}{\mu_B} \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)}$$

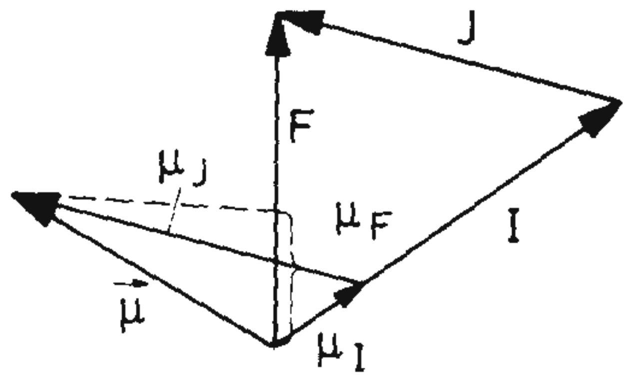


(b) strong field: $|H_{Zeeman}| \,\square\, |H_{HF}| \rightarrow \left[H,\vec{F}\right] \neq 0$

$$\Delta W = -m_I g_I \mu_N H_e + A m_I m_J - m_J g_J \mu_B H_e$$

Coupling of I and J governs the magnetic moment of the atom:





Hyperfine splitting for I=3/2, J=1/2

(Musiol /Ranft et al., Kern- und Elementarteilchenphysik)

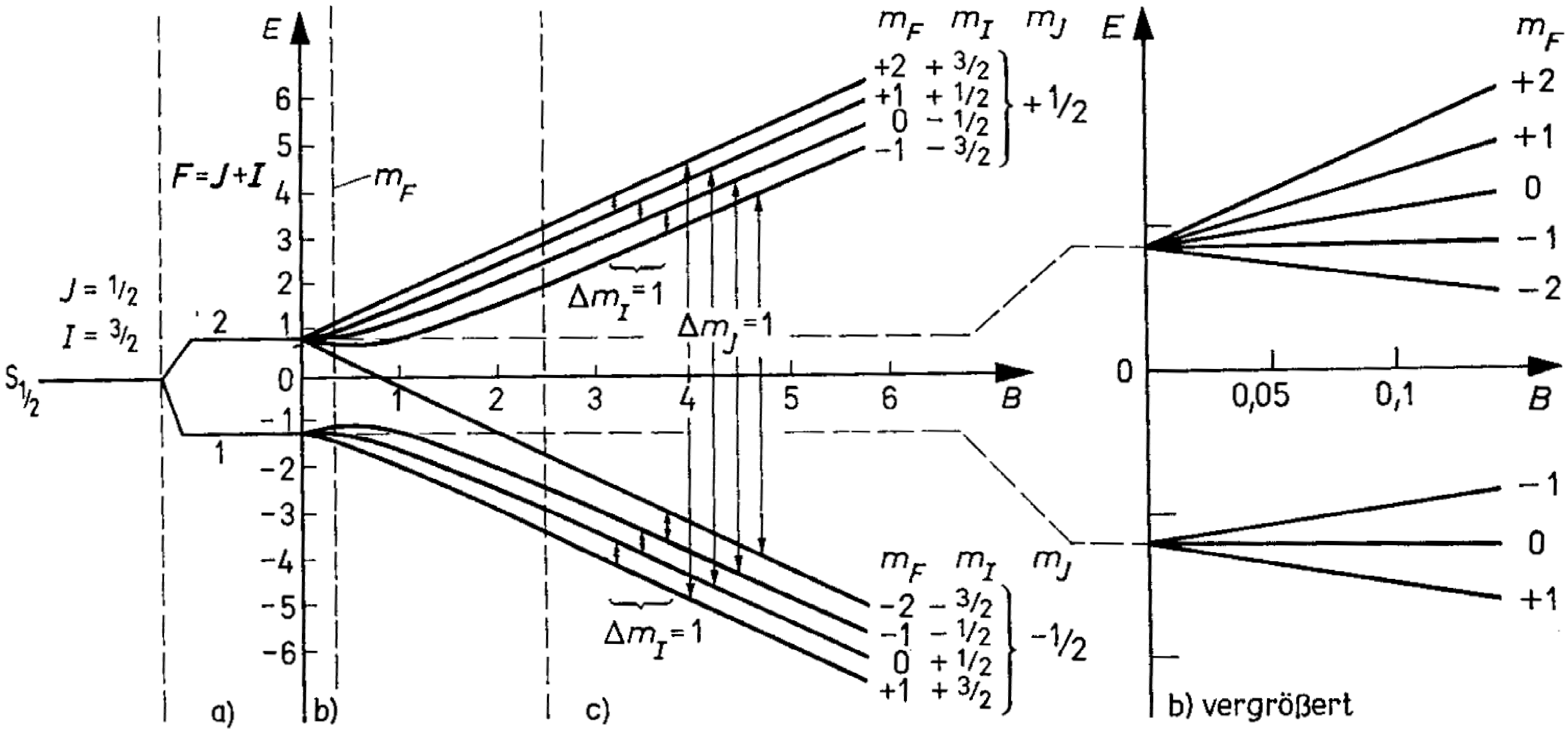
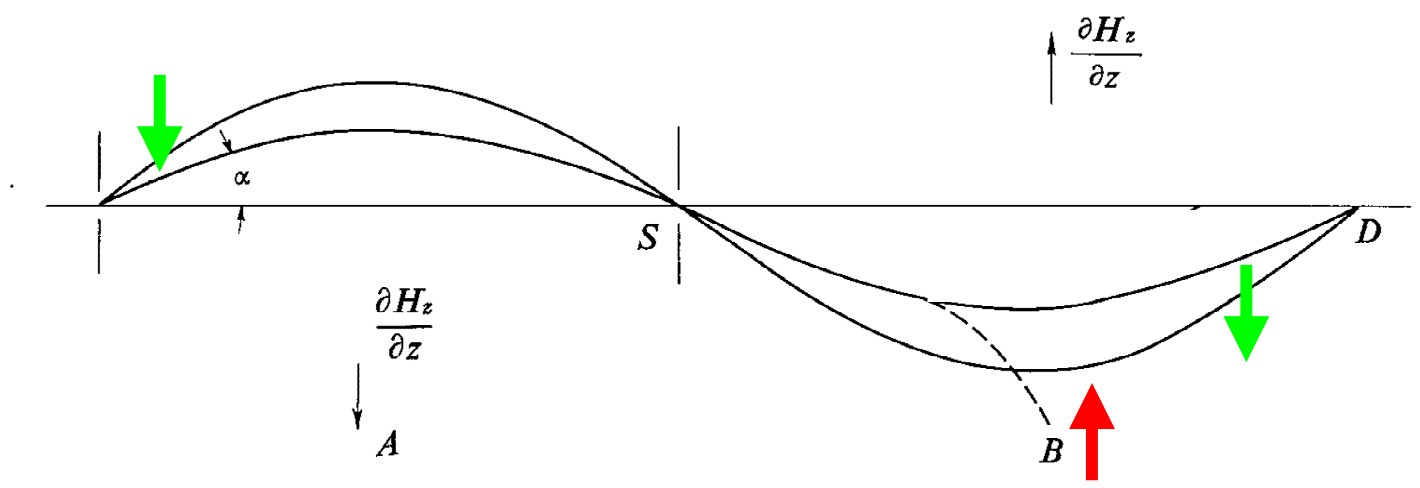
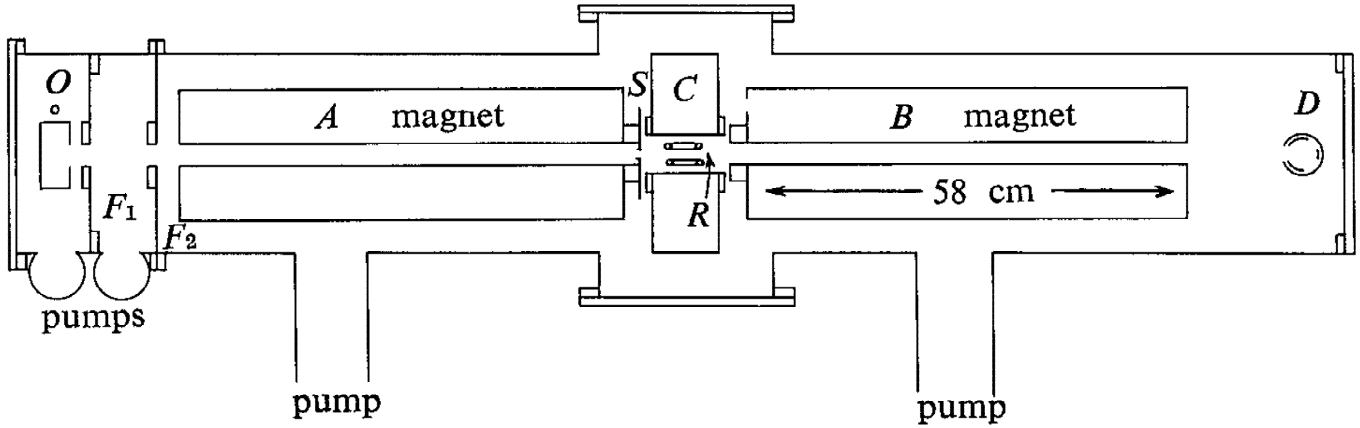


Abb. 6.28
HSF-Aufspaltung E als Funktion der magnetischen Induktion, beide in relativen Einheiten
a) ohne äußeres Magnetfeld, b) bei Zeeman-Effekt, c) bei Paschen-Back-Effekt

Rabi's atomic beam method





Example of a measurement using Rabi's method

An oscillatory field in C can induce a transition between the m states

$$E = g_I B_0 \mu_N m_I$$

$$\Delta E = g_I B_0 \mu_N$$

$$\text{Resonance Condition} \quad g_I B_0 \mu_N = h\nu$$

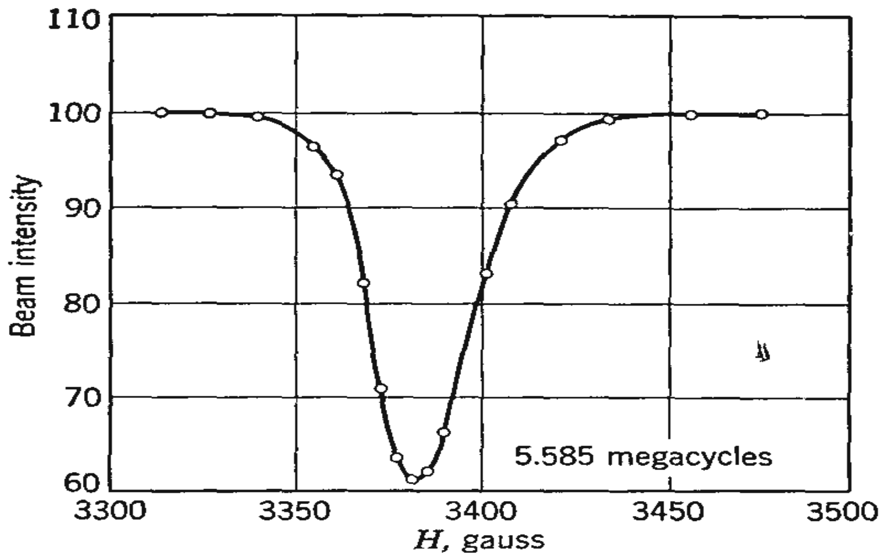


Figure 16.15 Measurement of resonance of ^{7}Li using apparatus of Figure 16.14. From I. I. Rabi et al., *Phys. Rev.* **53**, 318 (1938).