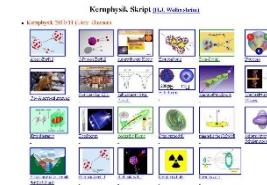


Outline: Nuclear rotation of odd-even nuclei

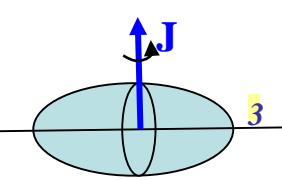
Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

web-page: <https://web-docs.gsi.de/~wolle/> and click on

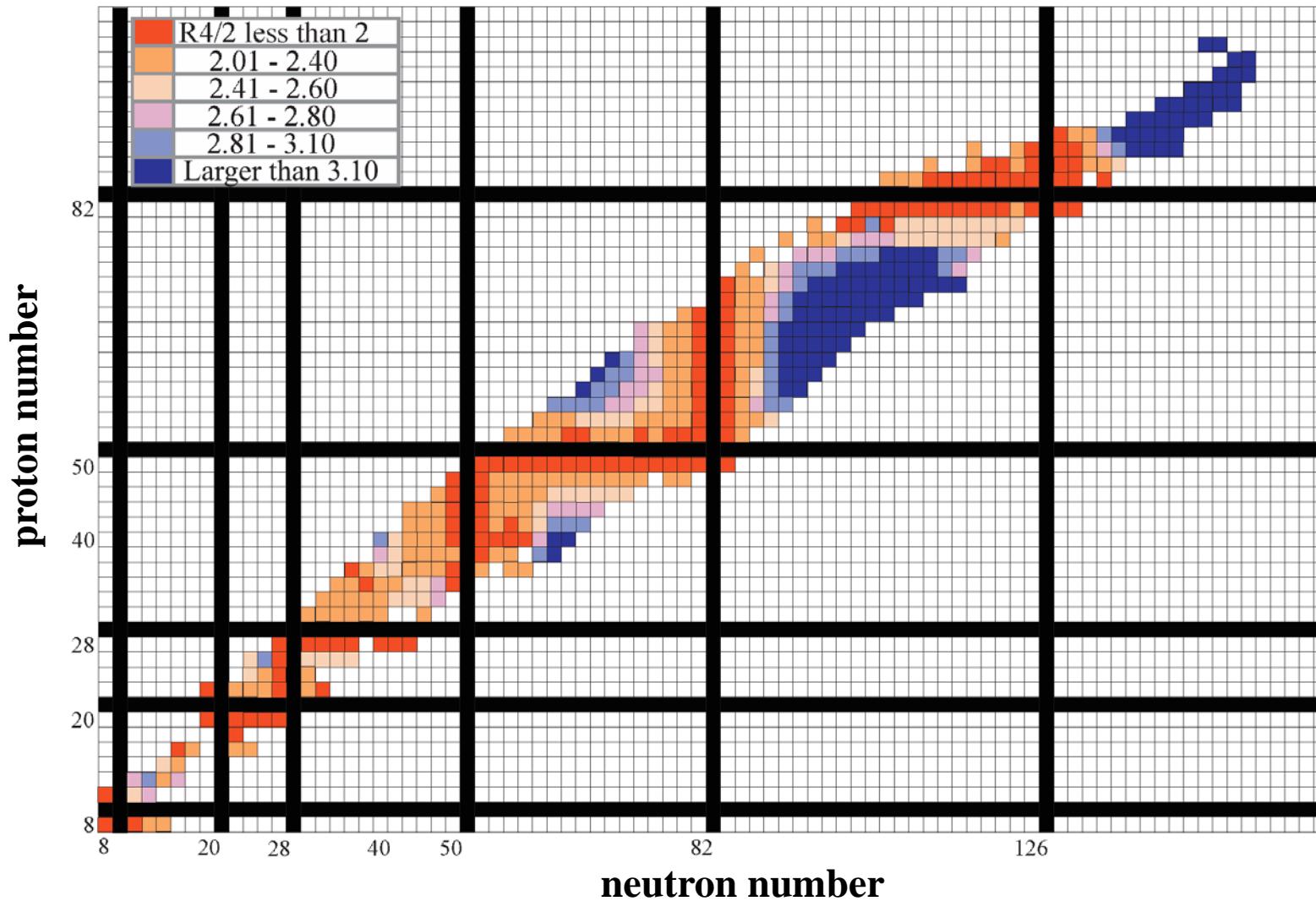


1. particle-rotor model
2. Euler angles
3. Example: ^{181}Ta
4. reduced transition probabilities



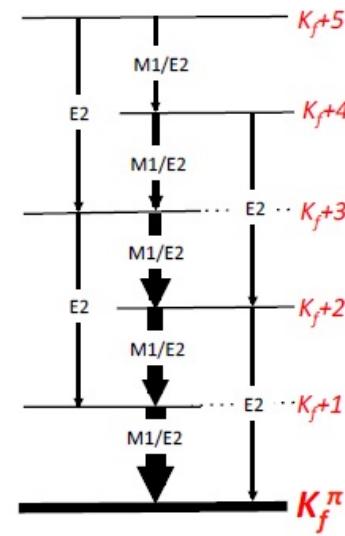
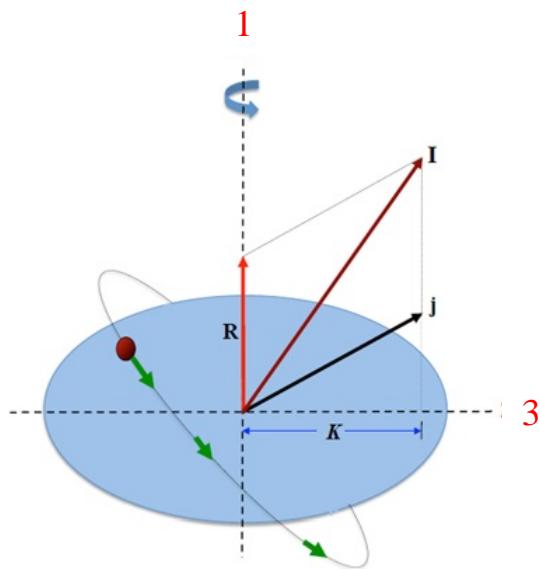
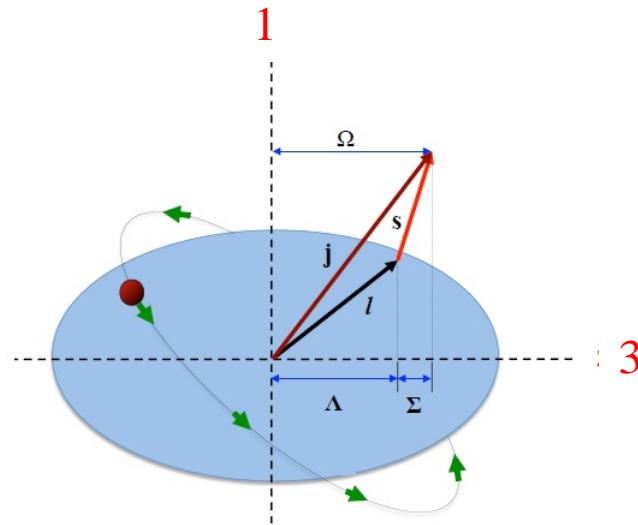
Broad perspective on structural evolution:

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

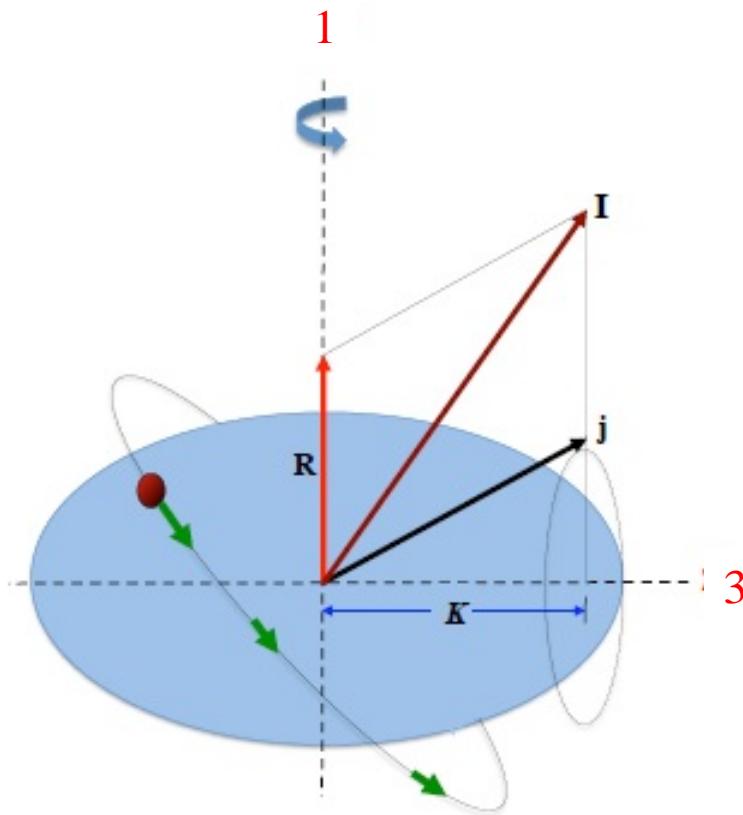


for even-even nuclei

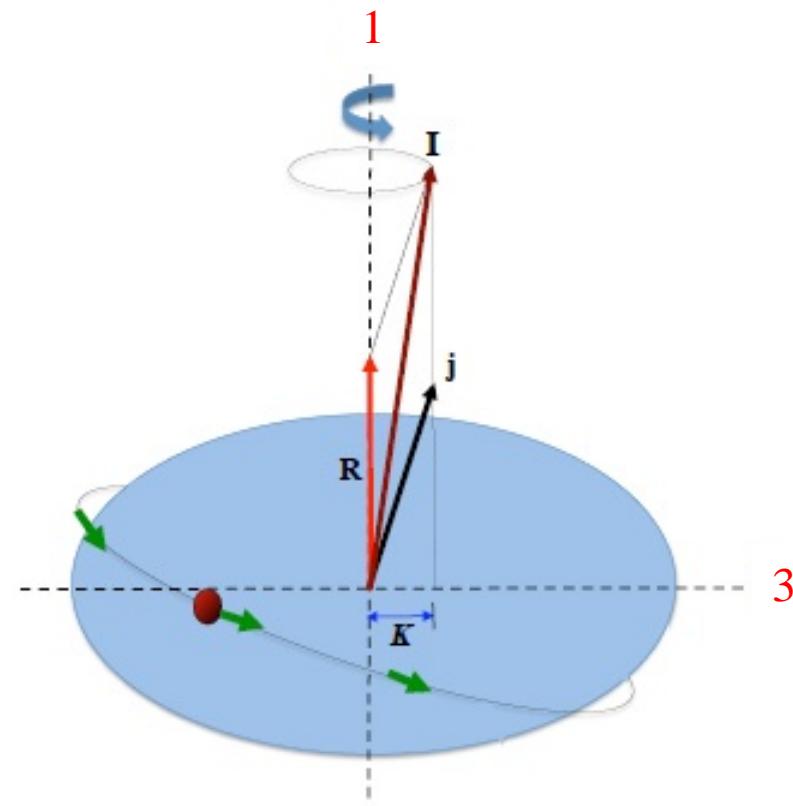
Odd-Even nuclei



Odd-Even nuclei: Coriolis coupling



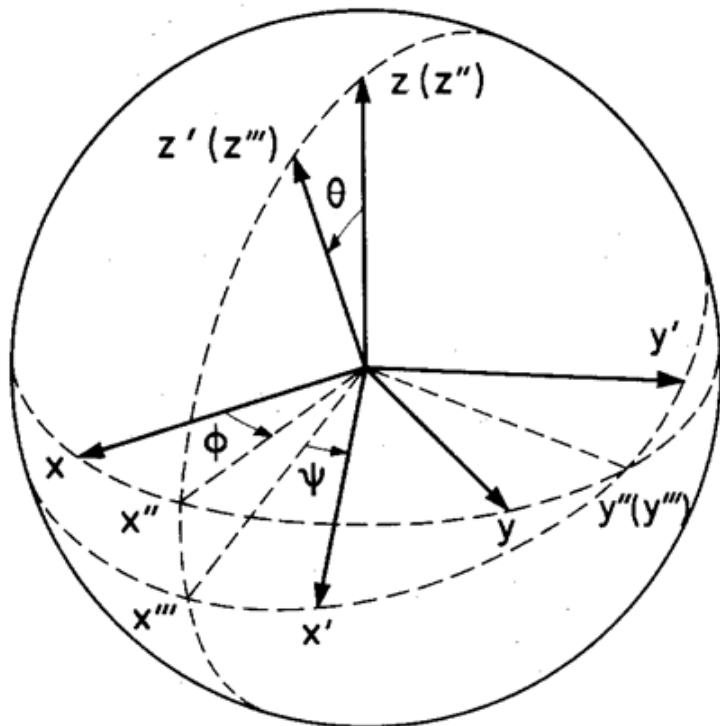
strong coupling



weak coupling
rotational alignment

The Euler angles

- It is important to recognize that for nuclei the intrinsic reference frame can have any orientation with respect to the lab reference frame as we can hardly control orientation of nuclei (although it is possible in some cases).
- One way to specify the mutual orientation of two reference frames of the common origin is to use Euler angles.



(x, y, z) axes of lab frame
(1,2,3) axes of intrinsic frame

- The rotation from (x,y,z) to (x',y',z') can be decomposed into three parts: a rotation by ϕ about the z axis to (x'',y'',z'') , a rotation of θ about the new y axis (y'') to (x''',y''',z''') , and finally a rotation of ψ about the new z axis (z''') .

Quantization

states : $|I, M, K\rangle$

laboratory axes : $[J_x, J_y] = i \cdot \hbar \cdot J_z$ and cyclic permutations

$$[J^2, J_k] = 0 \quad k = x, y, z$$

quantum numbers : $J_z \rightarrow \hbar \cdot M \quad J^2 \rightarrow \hbar^2 \cdot I(I + 1)$

body fixed axes : $[J_1, J_2] = i \cdot \hbar \cdot J_3$ and cyclic permutations

$$[J^2, J_i] = 0 \quad i = 1, 2, 3 \quad [J_z, J_3] = 0$$

quantum numbers : $J_3 \rightarrow \hbar \cdot K \quad J^2 \rightarrow \hbar^2 \cdot I(I + 1)$

Quantization

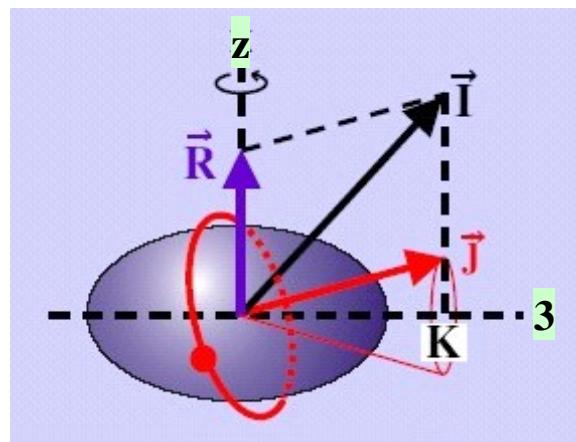
eigenstates: $| I, M, K >$

probability amplitude

for orientation of rotor: $\langle \psi, \theta, \phi | I, M, K \rangle = \left(\frac{2I+1}{8\pi^2} \right)^{1/2} D_{MK}^I(\psi, \theta, \phi)$

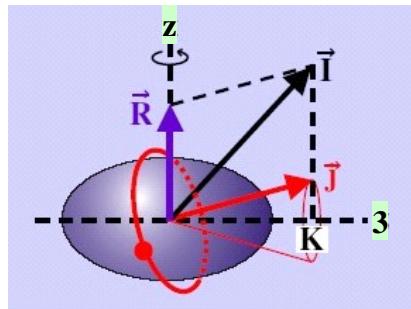
Wigner D – function

$$D_{MK}^I(\psi, \theta, \phi) = e^{iM\psi} d_{MK}^I(\theta) e^{iK\phi}$$





Rotational motion of a deformed nucleus



$$H_{rot} = \sum_{i=1}^3 \frac{\hat{R}_i^2}{2 \cdot \mathfrak{I}_i} = \frac{(\hat{R}^2 - \hat{R}_3^2)}{2 \cdot \mathfrak{I}_1} + \cancel{\frac{\hat{R}_3^2}{2 \cdot \mathfrak{I}_3}}$$

$$\mathfrak{I}_1 = \mathfrak{I}_2$$

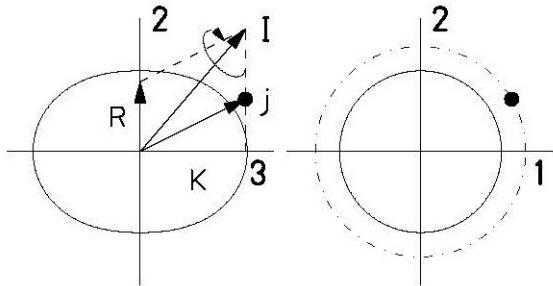
The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

States with projections K and $-K$ are degenerated

$$\Psi_{IMK} = \left(\frac{2 \cdot I + 1}{16 \cdot \pi^2} \right)^{1/2} \cdot [D_{MK}^I \cdot \chi_K + (-1)^{I-K} D_{M-K}^I \cdot \chi_{-K}]$$

Particle-rotor model

K+4
K+3
K+2
K+1
K



$$H_{rot} = \sum_{i=1}^3 \frac{\hat{R}_i^2}{2 \cdot \mathfrak{J}_i} = \frac{(\hat{R}^2 - \hat{R}_3^2)}{2 \cdot \mathfrak{J}_1} + \frac{\cancel{\hat{R}_3^2}}{2 \cdot \cancel{\mathfrak{J}_3}}$$

$\mathfrak{J}_1 = \mathfrak{J}_2$

The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

with $\vec{R} = \vec{I} - \vec{j}$

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{J}_0} \{(I_1 - j_1)^2 + (I_2 - j_2)^2\}$$

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{J}_0} (I^2 - I_3^2) + \frac{\hbar^2}{2\mathfrak{J}_0} (j_1^2 + j_2^2 - j_3^2) - \frac{\hbar^2}{2\mathfrak{J}_0} (j_+ I_- + j_- I_+)$$

$$I_{\pm} = I_1 \pm i \cdot I_2 \quad j_{\pm} = j_1 \pm i \cdot j_2$$

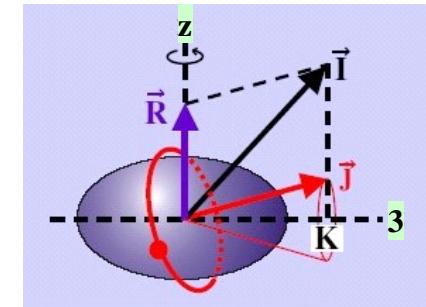
$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathfrak{J}} [I(I+1) - K^2 + \delta_{K,1/2} \cdot a \cdot (-1)^{I+1/2} (I+1/2)]$$

where a is the so-called *decoupling parameter*

Symmetries of rotating nuclei: Signature Quantum Number

- A rotation of 2π leaves the wave function unchanged, except for a possible phase factor (± 1), i.e.

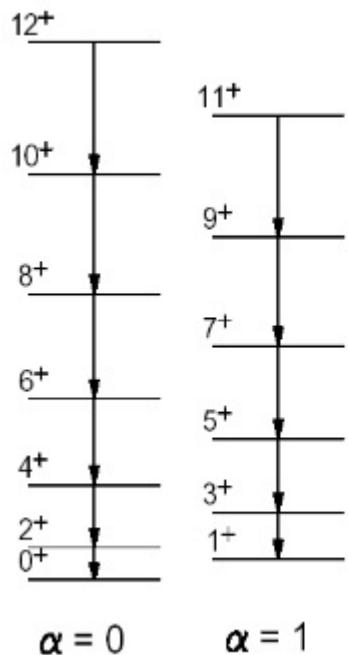
$$R_x^2(\pi)\Psi = r^2\Psi = (-1)^A\Psi \text{ with } r^2 = \pm 1$$



- The eigenvalues r of the rotation operator $R_x(\pi)$ called **signature**, are good quantum numbers, i.e. constant of the motion
- The **signature exponent quantum number α** is defined:

$$r = \exp(-i\pi\alpha)$$

Signature Quantum Numbers



- The signature is:

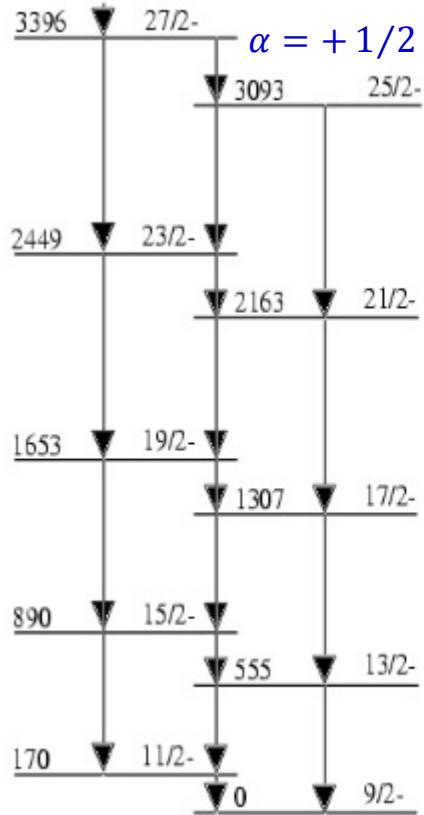
$$r = \pm 1 \text{ (even A)} \quad \alpha = 0, 1$$

$$r = \pm i \text{ (odd A)} \quad \alpha = \frac{1}{2}$$

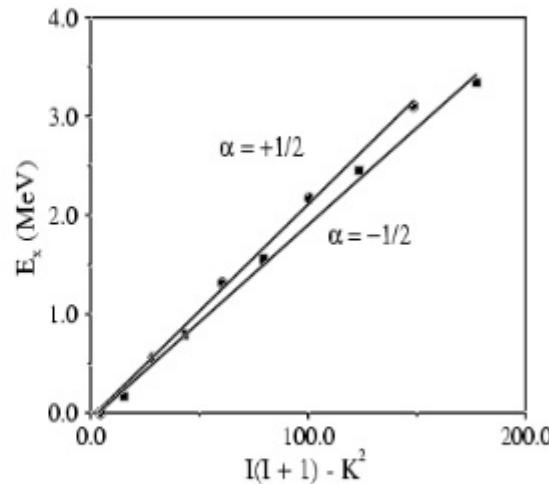
Only good quantum numbers: π and α

Signature Partners

$$\alpha = -1/2$$



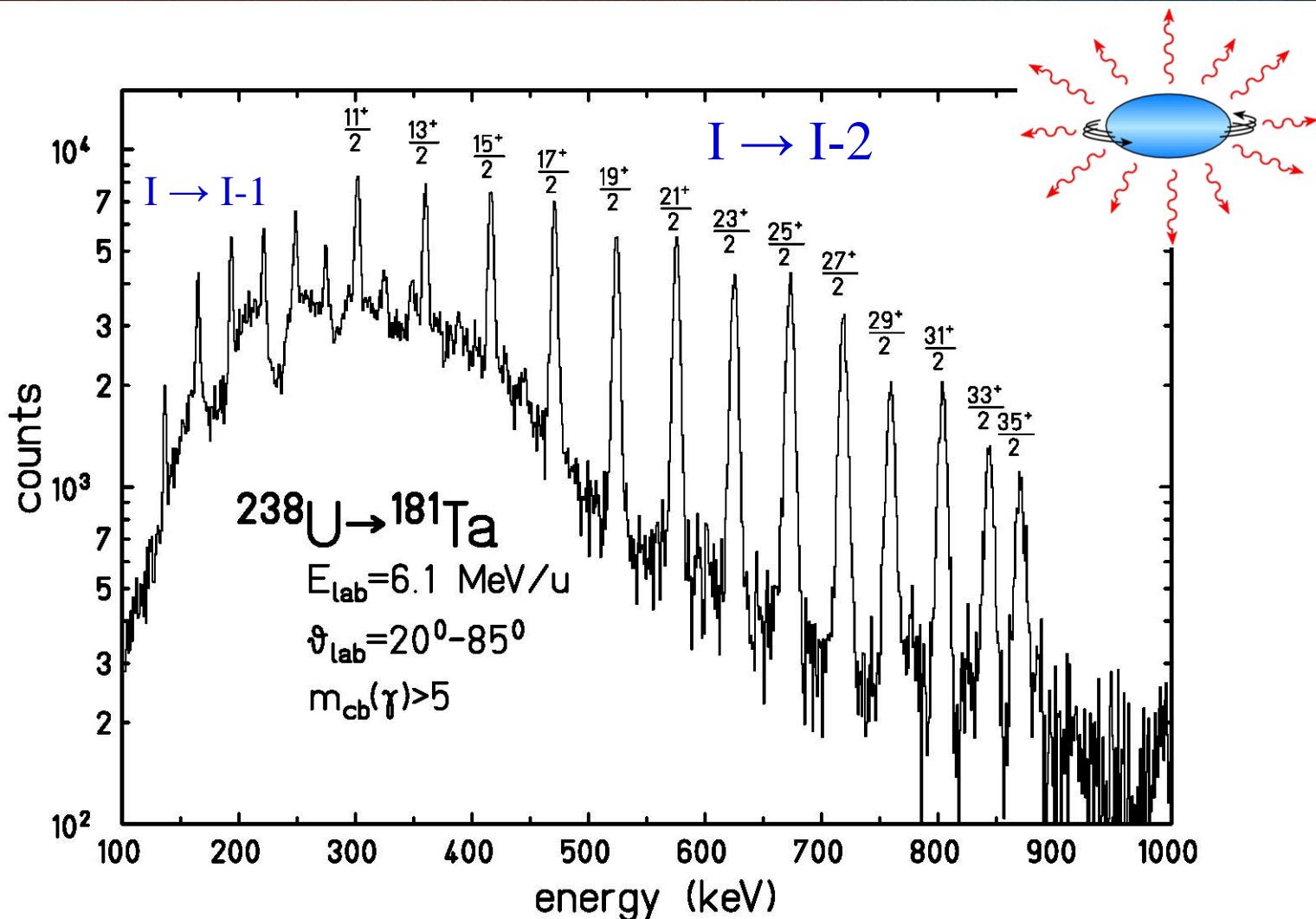
$$\alpha = +1/2$$



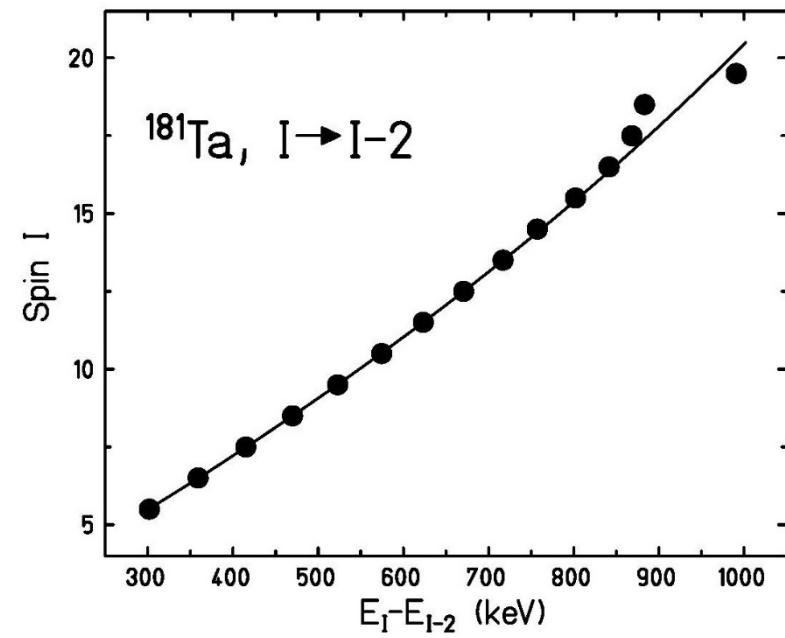
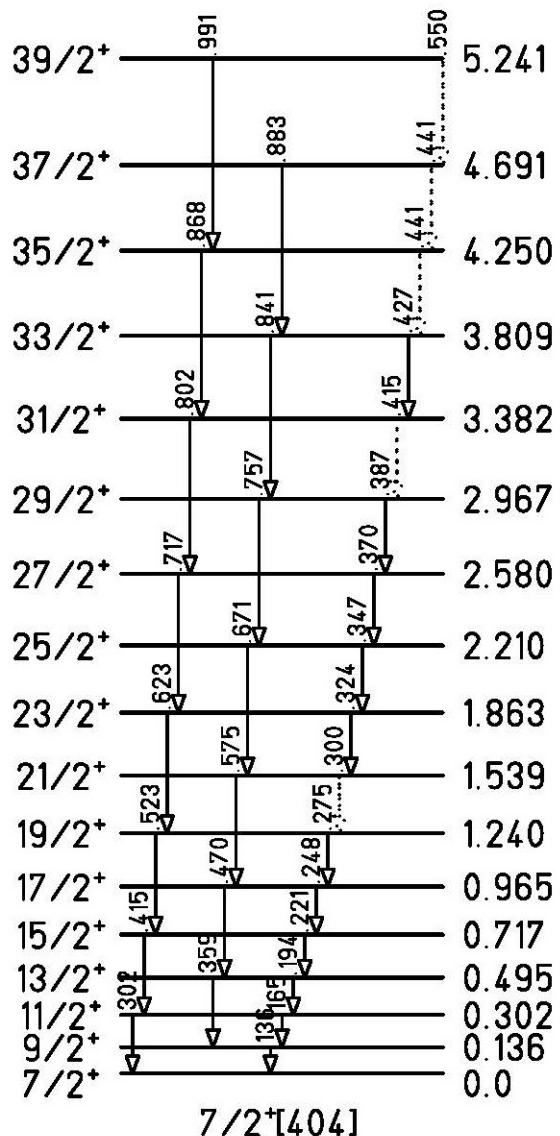
- Signature is a good quantum number at high spin
- A splitting between the $\alpha = 0, 1$ (even A) or $\alpha = \pm \frac{1}{2}$ (odd A) states gives rise to two distinct ‘signature partner’ bands

γ -rays from a deformed band in ^{181}Ta

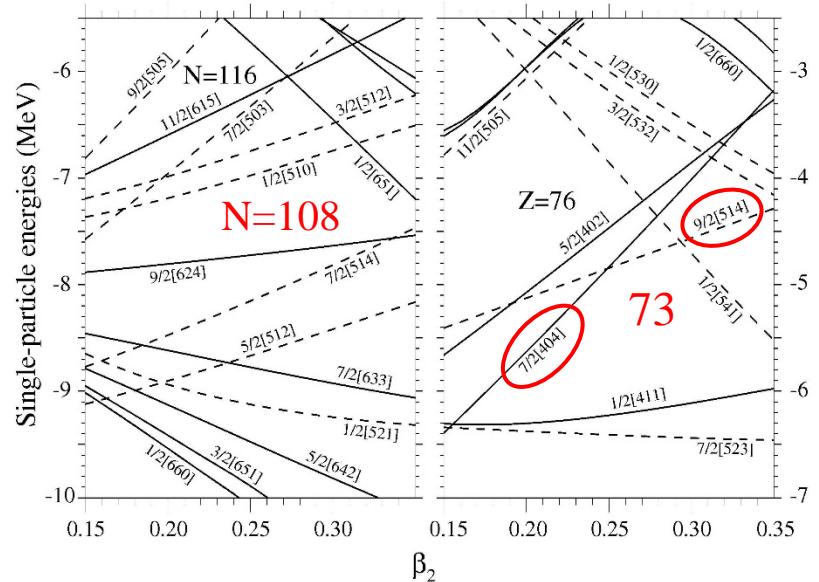
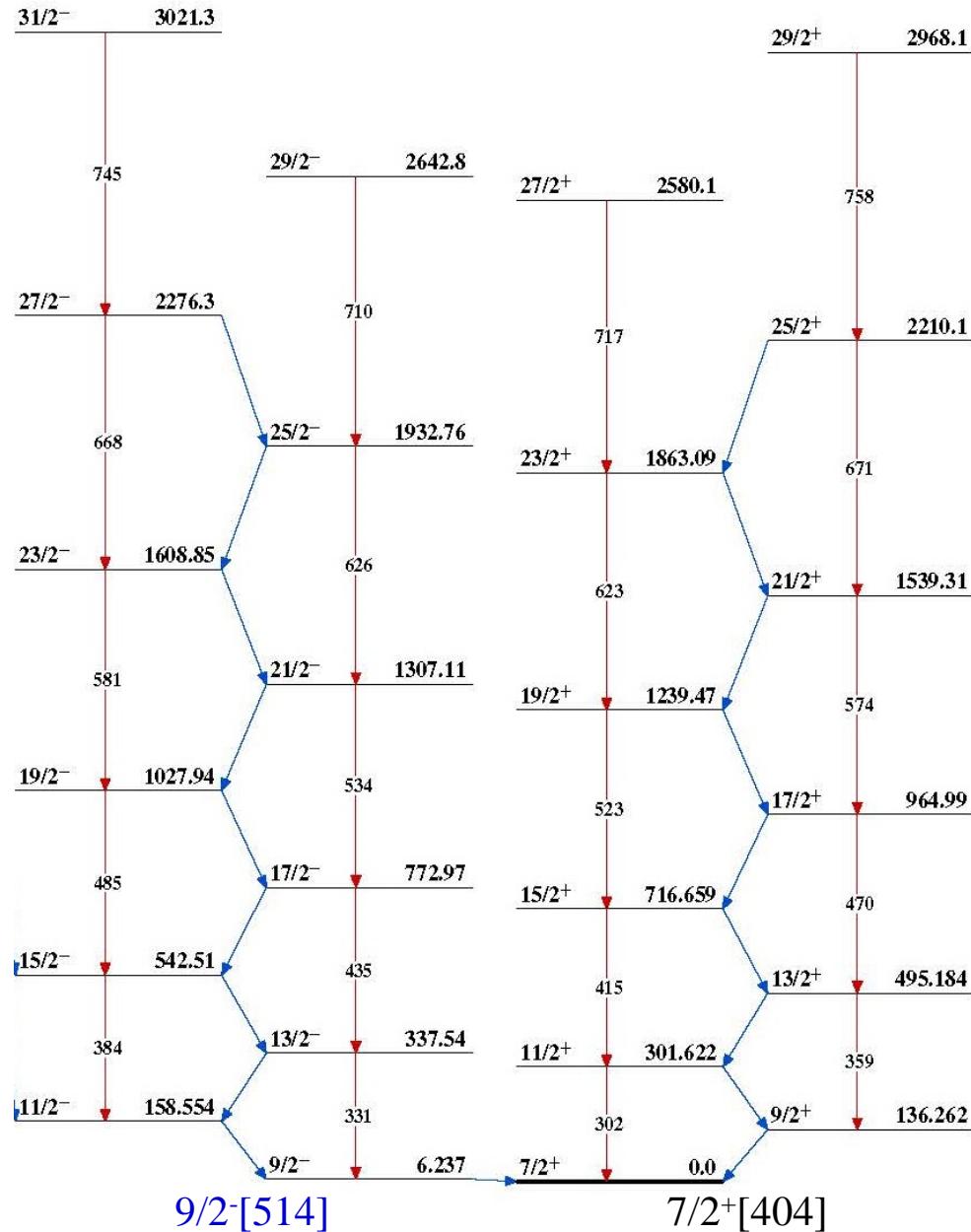
74	W	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189			
	4.22(24)	0.9 ms	7.3 ms	9.1 ms	4.10 ms	1.39 s	2.78 s	5.1 s	1.83 s	0.9 s	1.95 s	1.95 s	2.28 s	2.42 s	2.38 s	2.74 s	2.65 s	2.62 s	2.51 s	2.35 s	2.14 s	2.05 s	1.92 s	1.81 s	1.71 s	1.67 s	1.62 s	1.57 s	1.52 s	1.47 s	1.42 s	1.37 s				
	1.31(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*	1.25(10)	*				
73	Ta	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188		
	1.04(48)	1.01(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)	1.05(48)				
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188
	1.25(48)	0.39 s	1.25(48)	1.10(48)	0.38 s	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	1.25(48)	



^{181}Ta



Nilsson diagram of ^{181}Ta



Coriolis band mixing calculation for the $7/2^+[404]$ band in ^{181}Ta

The Hamiltonian H_{rot} is the diagonal part of the rotational Hamiltonian. The eigenvalues are assumed to be given by

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathfrak{J}} [I(I+1) - K^2 + \delta_{K,1/2} \cdot a \cdot (-1)^{I+1/2} (I+1/2)]$$

Off-diagonal terms are given by the Coriolis matrix elements

$$V_{K+1,K} = -\frac{\hbar^2}{2\mathfrak{J}} \sqrt{(I-K)(I+K+1)} \cdot \langle K+1|j_+|K\rangle$$

Results of Coriolis band mixing calculation for the ground-state rotational band in ^{181}Ta

State I^π	E_{calc} (keV)	Expansion coefficients						
		$\frac{1}{2}^+[422]$	$\frac{3}{2}^+[411]$	$\frac{3}{2}^+[402]$	$\frac{5}{2}^+[413]$	$\frac{5}{2}^+[402]$	$\frac{7}{2}^+[404]$	$\frac{9}{2}^+[404]$
$\frac{1}{2}^+$	0.0	0.0020	-0.0027	0.0	0.0417	-0.0304	0.9987	0.0
$\frac{3}{2}^+$	136.9	0.0040	-0.0053	0.0001	0.0630	-0.0459	0.9969	-0.0025
$\frac{11}{2}^+$	302.9	0.0065	-0.0085	0.0001	0.0818	-0.0594	0.9948	-0.0038
$\frac{13}{2}^+$	497.1	0.0093	-0.0123	0.0002	0.0995	-0.0720	0.9923	-0.0048
$\frac{15}{2}^+$	718.7	0.0127	-0.0167	0.0002	0.1167	-0.0840	0.9894	-0.0058
$\frac{17}{2}^+$	966.5	0.0163	-0.0216	0.0003	0.1335	-0.0955	0.9860	-0.0067
$\frac{19}{2}^+$	1239.3	0.0209	-0.0270	0.0	0.1501	-0.1067	0.9822	-0.0076
$\frac{21}{2}^+$	1535.7	0.0251	-0.0331	0.0004	0.1664	-0.1176	0.9781	-0.0084

Reduced transition probability

expectation value

$$\langle \hat{M}_{\lambda m}^{lab} \rangle = \int \Psi^* \hat{M}_{\lambda m}^{lab} \Psi d\tau$$

$$\hat{M}_{\lambda m}^{lab} = \sum_{m'} D_{mm'}^\lambda \hat{M}_{\lambda m'}^{intr}$$

wave function

$$\Psi_{IMK} = \sqrt{\frac{2I+1}{16 \cdot \pi^2}} \cdot [D_{MK}^I \cdot X_K + (-1)^{I-K} D_{M-K}^I \cdot X_{-K}]$$

$$\langle I_f M_f K | \hat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \frac{\sqrt{(2I_i + 1)(2I_f + 1)}}{8 \cdot \pi^2} \iiint D_{M_f K}^{I_f} X_K \sum_{m'=0} D_{mm'}^\lambda \hat{M}_{\lambda m'}^{intr} D_{M_i K}^{I_i} X_K d\tau$$

$$\iiint D_{M_1 M'_1}^{I_1} D_{M_2 M'_2}^{I_2} D_{M_3 M'_3}^{I_3} d\tau = \frac{8\pi^2}{2I_3 + 1} \cdot (I_1 I_2 M_1 M_2 | I_3 M_3) \cdot (I_1 I_2 M'_1 M'_2 | I_3 M'_3)$$

$$\langle I_f M_f K | \hat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \sqrt{\frac{2I_i + 1}{2I_f + 1}} \cdot (I_i \lambda M_i (M_f - M_i) | I_f M_f) \cdot (I_i \lambda K 0 | I_f K) \cdot \langle X_K | \hat{M}_{\lambda 0}^{intr} | X_K \rangle$$

Reduced transition probability

$$\langle I_f M_f K | \hat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \sqrt{\frac{2I_i + 1}{2I_f + 1}} \cdot (I_i \lambda M_i (M_f - M_i) | I_f M_f \rangle \cdot (I_i \lambda K 0 | I_f K \rangle \cdot \langle X_K | \hat{M}_{\lambda 0}^{intr} | X_K \rangle$$

Wigner-Eckart-Theorem (reduction of an expectation value):

$$\langle I_f M_f K | \hat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \frac{(I_i \lambda M_i (M_f - M_i) | I_f M_f \rangle}{\sqrt{2I_f + 1}} \cdot \langle I_f K | \hat{M}_{\lambda}^{lab} | I_i K \rangle$$

$$\boxed{\langle I_f K | M(E\lambda) | I_i K \rangle = \sqrt{2I_i + 1} (I_i \lambda K 0 | I_f K \rangle \cdot \langle X_K | \hat{M}_{\lambda 0}^{intr} | X_K \rangle)}$$

special case: E2 ($\lambda=2$) transition $I \rightarrow I-2$

$$\langle I-2, K | M(E2) | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

reduced transition probability:

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f K | M(E2) | I_i K \rangle|^2$$

Odd-even nucleus: ^{181}Ta

74	W	ν_{6550}	W158	0^+_1	W159	7.3^-	W160	9.1^-	W161	4.0^-	W162	1.3^-	W163	2.7^-	W164	5.1^-	W165	1.0^+_1	W166	1.0^+_2	W167	1.0^+_3	W168	1.0^+_4	W169	1.0^+_5	W170	2.4^+_1	W171	2.3^+_1	W172	4.0^+_1	W173	7.0^+_1	W174	3.1^+_1	W175	25.2^+_1	W176	7.5^+_1	W177	13.5^+_1	W178	21.4^+_1	W179	20.0^+_1	W180	1.0^+_1	W181	1.1^+_1	W182	1.1^+_2	W183	1.1^+_3	W184	49.4^+_1	W185	75.1^+_1	W186	69.4^+_1	W187	23.7^+_1	W188	11.5^+_1	W189	10.0^+_1	W190	10.0^+_1								
73	Ta	100^+	Ta156	1.0^+_1	Ta157	1.0^+_2	Ta158	1.0^+_3	Ta159	1.0^+_4	Ta160	1.0^+_5	Ta161	1.0^+_6	Ta162	1.0^+_7	Ta163	1.0^+_8	Ta164	1.0^+_9	Ta165	1.0^+_10	Ta166	1.0^+_11	Ta167	1.0^+_12	Ta168	1.0^+_13	Ta169	1.0^+_14	Ta170	1.0^+_15	Ta171	1.0^+_16	Ta172	1.0^+_17	Ta173	1.0^+_18	Ta174	1.0^+_19	Ta175	1.0^+_20	Ta176	1.0^+_21	Ta177	1.0^+_22	Ta178	1.0^+_23	Ta179	1.0^+_24	Ta180	1.0^+_25	Ta181	1.0^+_26	Ta182	1.0^+_27	Ta183	1.0^+_28	Ta184	49.4^+_1	Ta185	75.1^+_1	Ta186	69.4^+_1	Ta187	23.7^+_1	Ta188	11.5^+_1	Ta189	10.0^+_1	Ta190	10.0^+_1				
72	Hf	100^+	Hf154	0^+_1	Hf155	0^+_2	Hf156	0^+_3	Hf157	0^+_4	Hf158	0^+_5	Hf159	0^+_6	Hf160	0^+_7	Hf161	0^+_8	Hf162	0^+_9	Hf163	0^+_10	Hf164	0^+_11	Hf165	0^+_12	Hf166	0^+_13	Hf167	0^+_14	Hf168	0^+_15	Hf169	0^+_16	Hf170	0^+_17	Hf171	0^+_18	Hf172	0^+_19	Hf173	0^+_20	Hf174	0^+_21	Hf175	0^+_22	Hf176	0^+_23	Hf177	0^+_24	Hf178	0^+_25	Hf179	0^+_26	Hf180	0^+_27	Hf181	0^+_28	Hf182	0^+_29	Hf183	0^+_30	Hf184	49.4^+_1	Hf185	75.1^+_1	Hf186	69.4^+_1	Hf187	23.7^+_1	Hf188	11.5^+_1	Hf189	10.0^+_1	Hf190	10.0^+_1

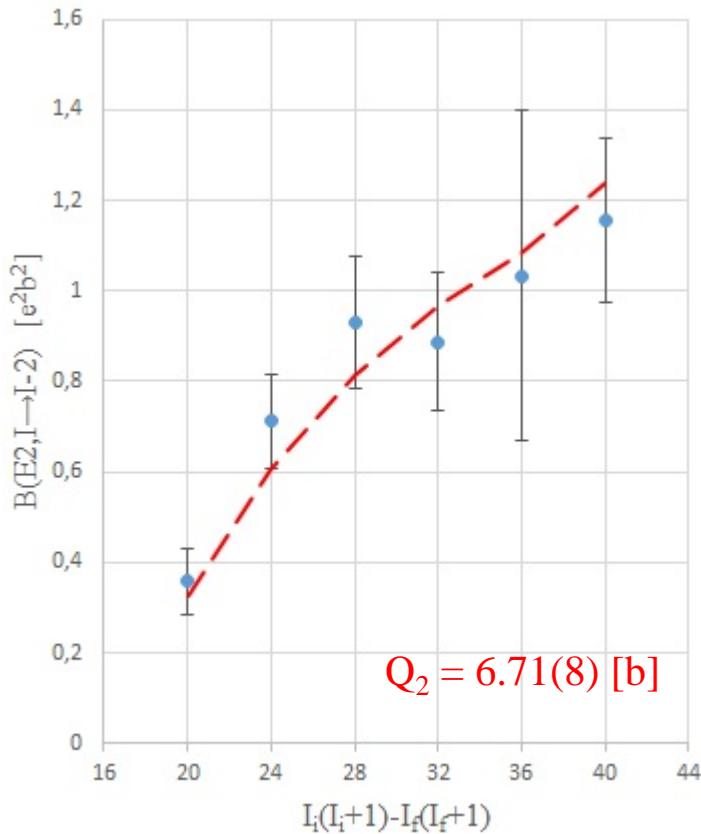
special case: E2 transition I→I-2

$$\langle I-2, K | M(E2) | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

reduced transition probability:

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f K | M(E2) | I_i K \rangle|^2$$

$$\frac{Q_2(9/2^-)}{Q_2(7/2^+)} = (0.9681 \pm 0.0002)$$



Matrix elements

$$\langle I-2, K | M(E2) | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

$$\langle I-1, K | M(E2) | I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3 \cdot (I+K) \cdot (I-K) \cdot K^2}{(I-1) \cdot I \cdot (I+1)}} \cdot Q_2 e$$

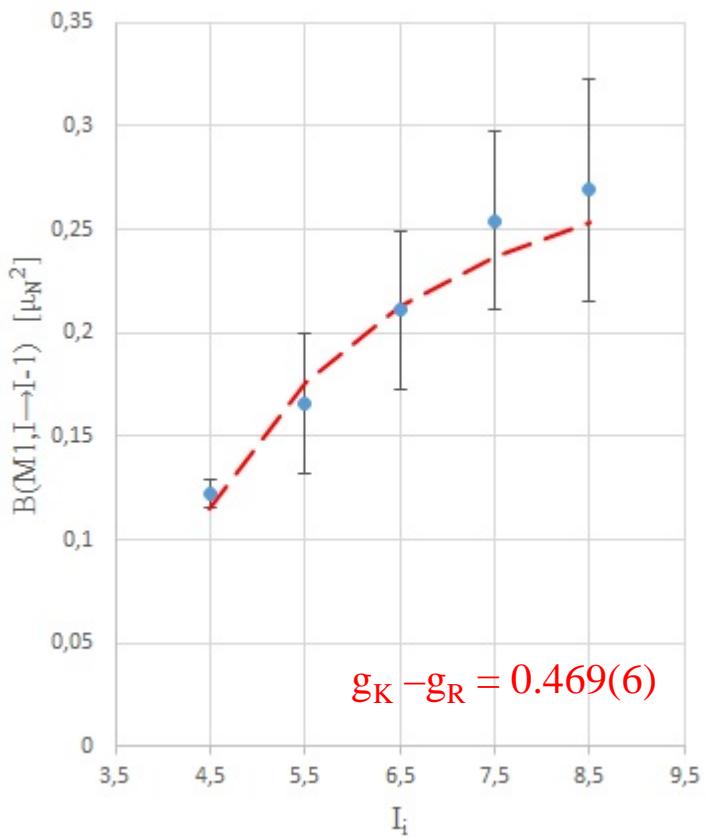
$$\langle I, K | M(E2) | I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{2I+1}{(2I-1) \cdot I \cdot (I+1) \cdot (2I+3)}} \cdot (I^2 - 3K^2 + I) \cdot Q_2 e$$

Odd-even nucleus: ^{181}Ta

74	W	^{159}W	^{158}W	^{159}W	^{160}W	^{161}W	^{162}W	^{163}W	^{164}W	^{165}W	^{166}W	^{167}W	^{168}W	^{169}W	^{170}W	^{171}W	^{172}W	^{173}W	^{174}W	^{175}W	^{176}W	^{177}W	^{178}W	^{179}W	^{180}W	^{181}W	^{182}W	^{183}W	^{184}W	^{185}W	^{186}W	^{187}W	^{188}W			
73	Ta	^{156}Ta	^{157}Ta	^{158}Ta	^{159}Ta	^{160}Ta	^{161}Ta	^{162}Ta	^{163}Ta	^{164}Ta	^{165}Ta	^{166}Ta	^{167}Ta	^{168}Ta	^{169}Ta	^{170}Ta	^{171}Ta	^{172}Ta	^{173}Ta	^{174}Ta	^{175}Ta	^{176}Ta	^{177}Ta	^{178}Ta	^{179}Ta	^{180}Ta	^{181}Ta	^{182}Ta	^{183}Ta	^{184}Ta	^{185}Ta	^{186}Ta	^{187}Ta	^{188}Ta		
72	Hf	^{154}Hf	^{155}Hf	^{156}Hf	^{157}Hf	^{158}Hf	^{159}Hf	^{160}Hf	^{161}Hf	^{162}Hf	^{163}Hf	^{164}Hf	^{165}Hf	^{166}Hf	^{167}Hf	^{168}Hf	^{169}Hf	^{170}Hf	^{171}Hf	^{172}Hf	^{173}Hf	^{174}Hf	^{175}Hf	^{176}Hf	^{177}Hf	^{178}Hf	^{179}Hf	^{180}Hf	^{181}Hf	^{182}Hf	^{183}Hf	^{184}Hf	^{185}Hf	^{186}Hf	^{187}Hf	^{188}Hf
	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188			
	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188			

$$\langle I-1, K | M(M1) | I, K \rangle = -\sqrt{\frac{3}{4\pi}} \sqrt{\frac{(I+K)(I-K)}{I}} \cdot K \cdot (g_K - g_R) [1 + \delta_{K,1/2} (-1)^{I+1/2} b_0] \mu_N$$

The quantity b_0 depends on the magnetic decoupling parameter



M. Loewe; dissertation

$$\mu(7/2^+) = 2.3705 \pm 0.0007$$

$$\mu = \frac{K}{I+1} \cdot (g_K - g_R) \cdot K + g_R \cdot I$$

$$g_R = 0.313(5)$$

$$g_K = 0.782(2)$$

$$\mu(9/2^-) = 5.28 \pm 0.09$$

$$^{9/2^-}(g_K - g_R) = \frac{22}{81} \left(^{9/2^-} \mu - ^{7/2^+} \mu \frac{9}{7} + ^{7/2^+} (g_K - g_R) \frac{7}{2} \right)$$

$$^{9/2^-}(g_K - g_R) \approx ^{7/2^+} (g_K - g_R) \frac{77}{81} + 0.606 = 1.052$$

Odd-even nucleus: ^{181}Ta

74	W	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190		
73	Ta	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188	116	
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188
71																																				

$13/2^+$ — $0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

$11/2^+$ — $0.302 \quad \tau = 23.1 \pm 4.3 \text{ ps}$

$$\tau = T_{1/2} / \ln 2$$

$9/2^+$ — $0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$

$7/2^+$ — $0.0 \quad \text{E2 + M1}$

^{181}Ta

$$\delta_{N \rightarrow M}(\lambda) = \left\{ \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\lambda+1} \right\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(\lambda) || I_N \rangle$$

$$\delta_{N \rightarrow M}(E2) = \{1.225 \cdot 10^{13} \cdot E_{\gamma}^5 (\text{MeV})^5\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(E2) || I_N \rangle$$

$$\delta_{N \rightarrow M}(M1) = \{1.758 \cdot 10^{13} \cdot E_{\gamma}^3 (\text{MeV})^3\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(M1) || I_N \rangle$$

$$\varepsilon_{N \rightarrow M}^2(\ell) = \delta_{N \rightarrow M}^2(\ell) \cdot \alpha_{N \rightarrow M}(\ell)$$

T. Inamura et al., Nucl. Phys. A270 (1976) 255

conversion coefficient.: bricc.anu.edu.au

Odd-even nucleus: ^{181}Ta

74 Ta 73 Hf	W W155 W158 W159 W160 W161 W162 W163 W164 W165 W166 W167 W168 W169 W170 W171 W172 W173 W174 W175 W176 W177 W178 W179 W180 W181 W182 W183 W184 W185 W186 W187 W188 W189 W190

72

$$13/2^+ \quad 0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$$

$$\tau = \left\{ \sum_K \sum_\ell [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

$$11/2^+ \quad 0.302 \quad \tau = 23.1 \pm 4.3 \text{ ps}$$

$$\tau = T_{1/2} / \ln 2$$

$$9/2^+ \quad 0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$$

$$7/2^+ \quad 0.0$$

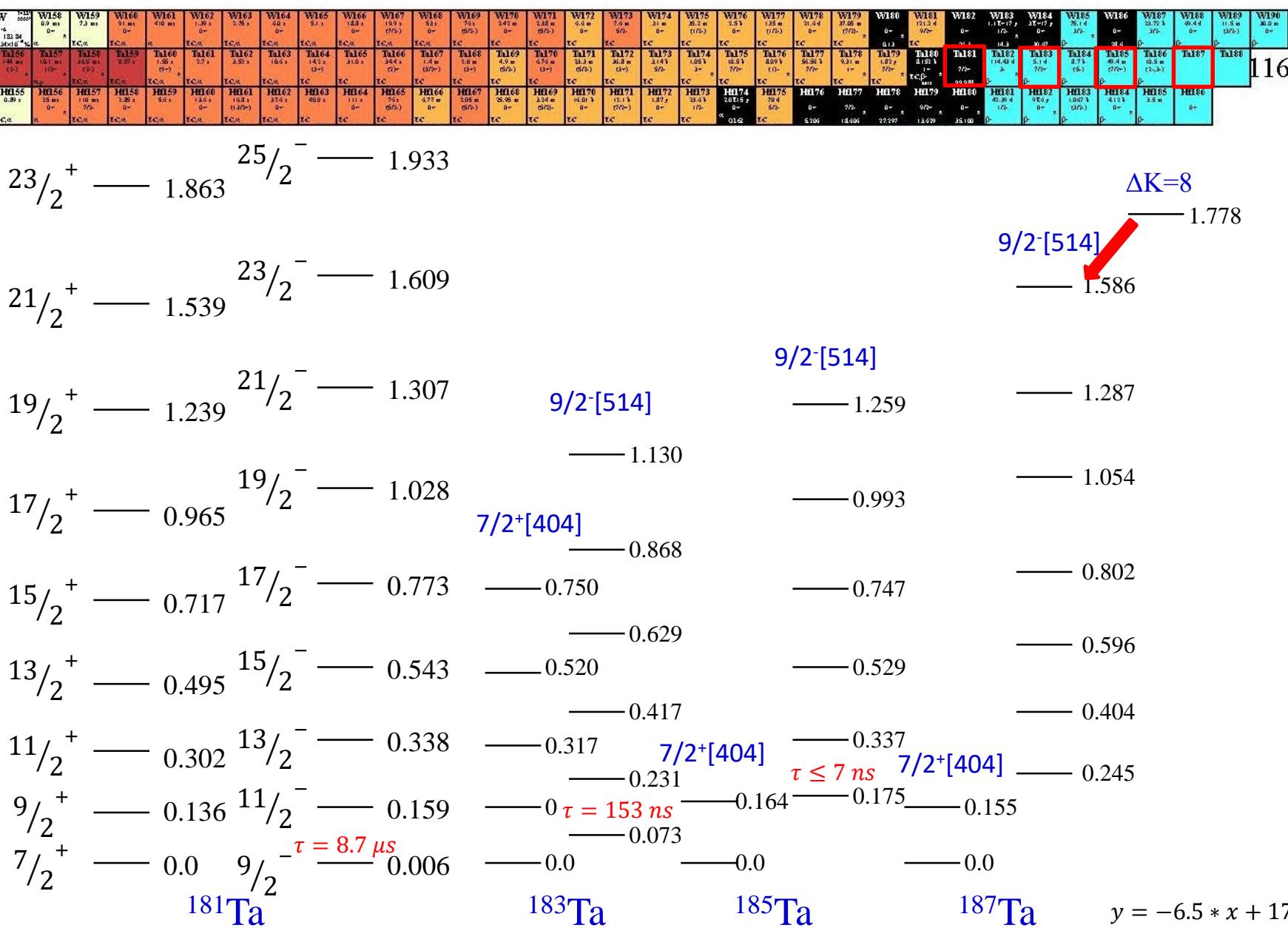
^{181}Ta	Spin I	E_γ (MeV)	$(2I+1)^{-1/2}$	$\langle I-1/M(0)/I \rangle$	delta	α_T	ε^2	τ (ps)
	9/2	0.1365	0.3162	-3.899 (E2)	-29594.	1.1	$9.98 \cdot 10^8$	533
				1.103 (M1)	73591	1.8	$9.75 \cdot 10^9$	58.7
	11/2	0.1654	0.2887	-4.291 (E2)	-48238	0.6	$1.33 \cdot 10^9$	274
				1.413 (M1)	115044	1.0	$1.38 \cdot 10^{10}$	32.6
	11/2	0.3017	0.2887	1.977 (E2)	99868	0.1	$8.08 \cdot 10^8$	24.1

T. Inamura et al., Nucl. Phys. A270 (1976) 255

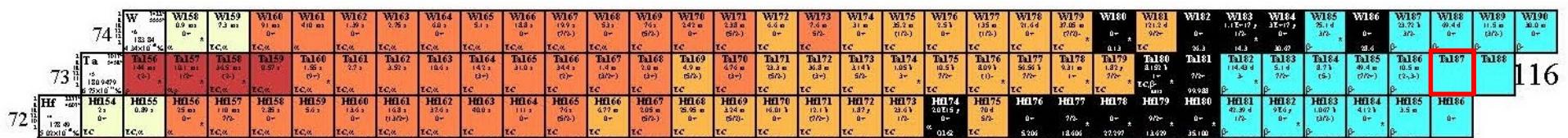
Oddproton-evenneutron nuclei

		μ	$(g_K - g_R)$	$B(M1)W.u.$	$(g_K - g_R)$
^{153}Eu	$5/2^+$	+1.5324(3)	0.551	0.00608(28)	0.185
^{159}Tb	$3/2^+$	+2.014(4)	1.556	0.173(8)	1.471
^{165}Ho	$7/2^-$	+4.177(5)	1.012	0.275(14)	0.973
^{169}Tm	$1/2^+$	-0.2316(15)		0.0342(8)	
^{175}Lu	$7/2^+$	+2.2327(11)	0.298	0.0354(14)	0.349
^{181}Ta	$7/2^+$	+2.3705(7)	0.352	0.068(4)	0.484
^{185}Re	$5/2^+$	+3.1871(3)	1.217	0.28(5)	1.252
^{187}Re	$5/2^+$	+3.2197(3)	1.242	0.260(18)	1.206

Ta-nuclei: level schemes



^{187}Ta level scheme



expected lifetimes

$$\Delta K=8$$

$$(25/2^-) \text{--- } 1.778 \quad \tau = 10.5(13) \text{ s}$$

$$9/2^-[514] \quad \quad \quad$$

$$(21/2^-) \text{--- } 1.586 \quad \tau = 0.6 \text{ ps}$$

$$(19/2^-) \text{--- } 1.287 \quad \tau = 1.1 \text{ ps}$$

$$(17/2^-) \text{--- } 1.054 \quad \tau = 1.1 \text{ ps}$$

$$(15/2^-) \text{--- } 0.802 \quad \tau = 2.2 \text{ ps}$$

$$(13/2^-) \text{--- } 0.596 \quad \tau = 3.4 \text{ ps}$$

$$(11/2^-) \text{--- } 0.404 \quad \tau = 7.9 \text{ ps}$$

$$7/2^+[404] \quad (9/2^-) \text{ --- } 0.245 \quad \tau \sim 40 \text{ ns}$$

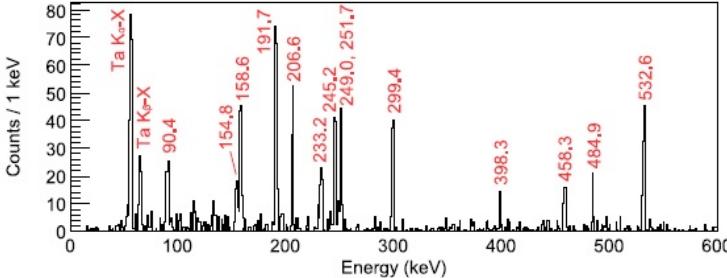
$$(9/2^+) \text{ --- } 0.155$$

$$(7/2^+) \text{ --- } 0.0$$

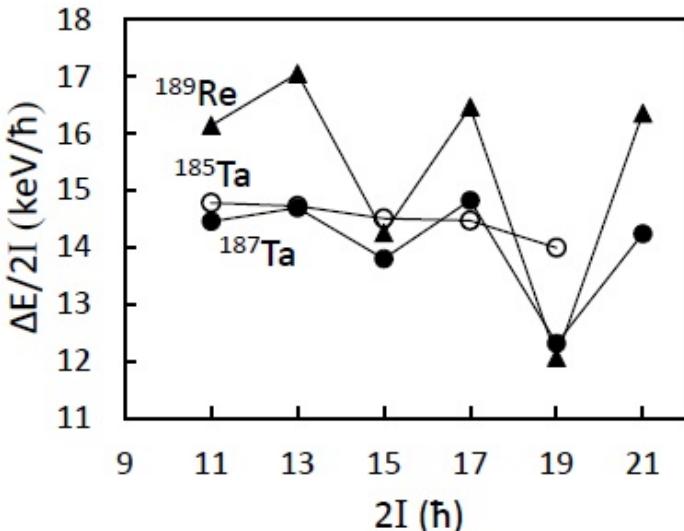
^{187}Ta P.M. Walker; Phys. Rev. Lett. 125 192505

$I \rightarrow I - 1$

$I \rightarrow I - 2$



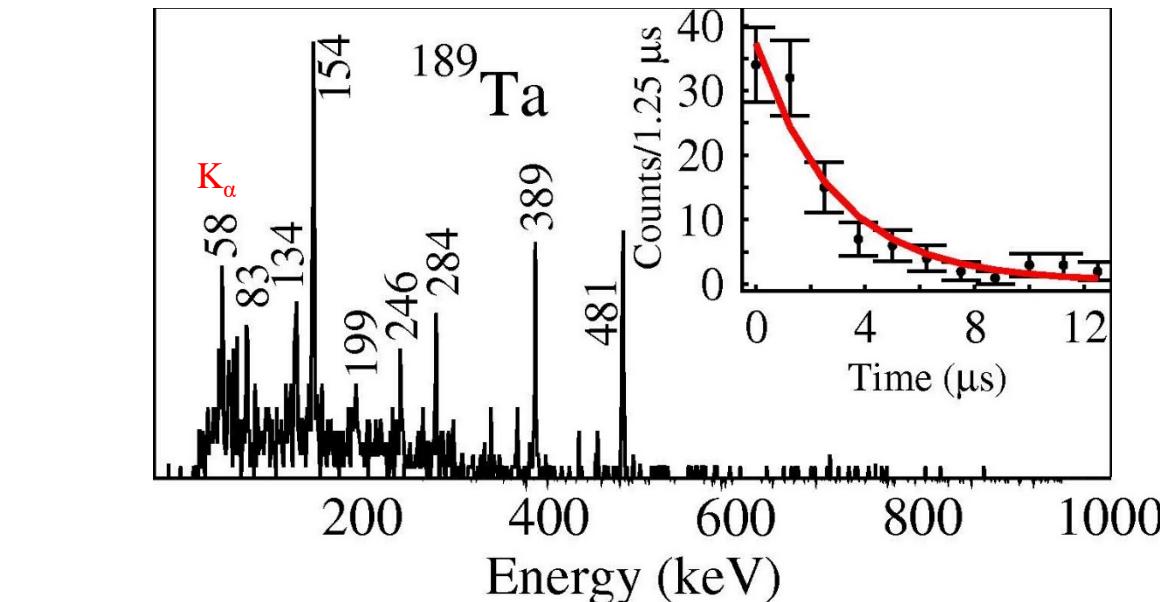
Sum of $\gamma\gamma$ -coincidence energy spectra



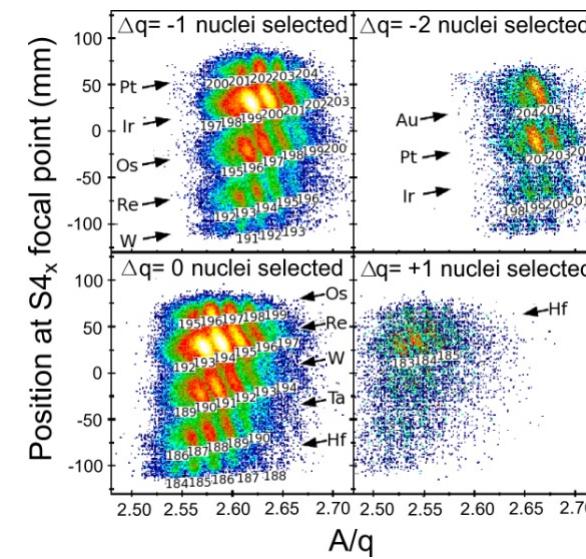
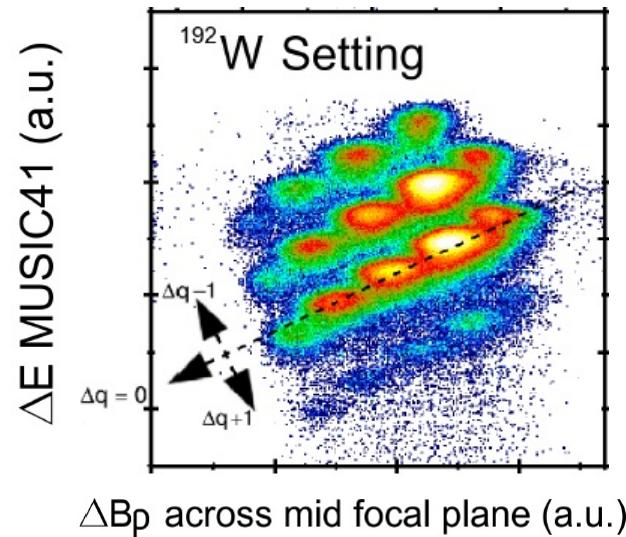
Signature splitting versus angular momentum

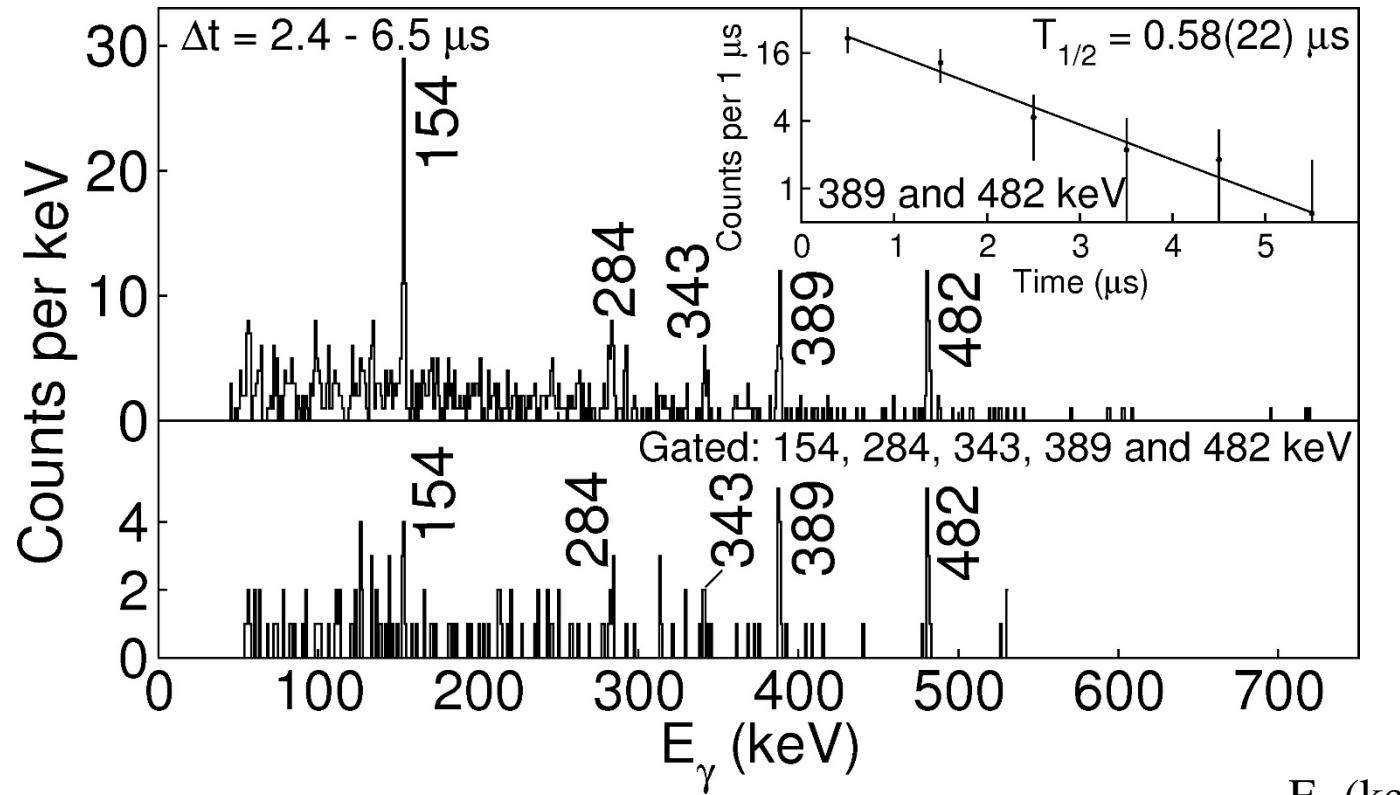
$$y = -6.5 * x + 17$$

Experimental information on ^{189}Ta



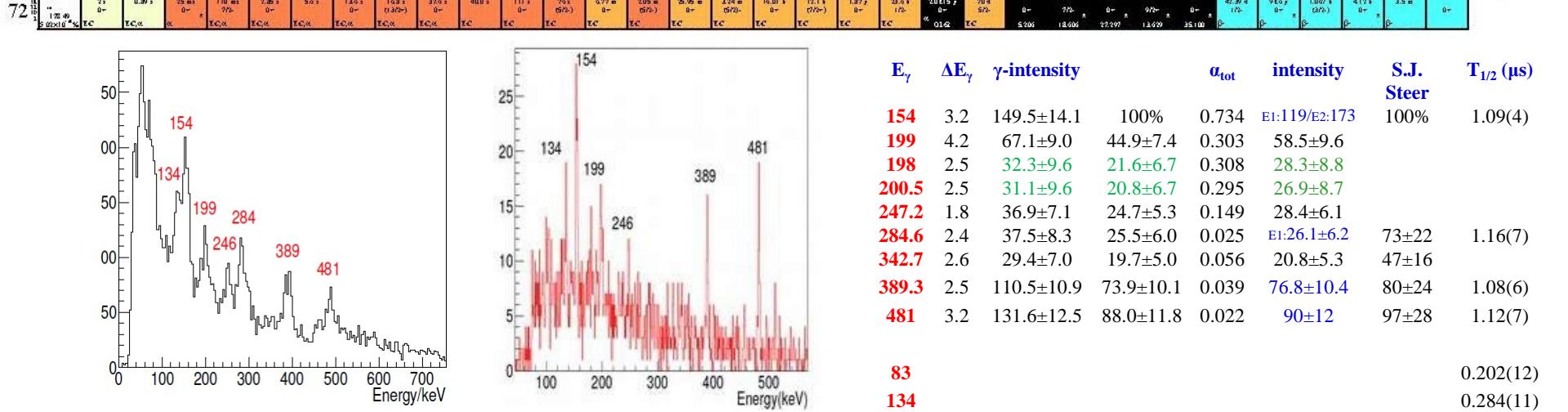
^{208}Pb primary beam



Experimental information on ^{189}Ta 

E_γ (keV)	intensity
153.9	100(19)
283.7	73(17)
342.5	47(13)
388.7	80(19)
481.6	97(21)

Experimental information on ^{189}Ta

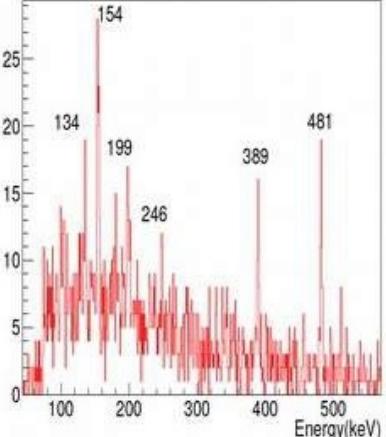
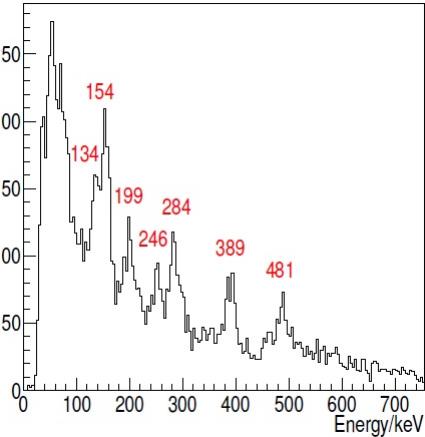


gate

481	154	284	389	-
389	154	284	-	481
284	154	-	389	481
343	154	199	246	-
246	154	199	-	343 370
199	154	-	389	481
154	134	154	246	343
134			284	389
			481	

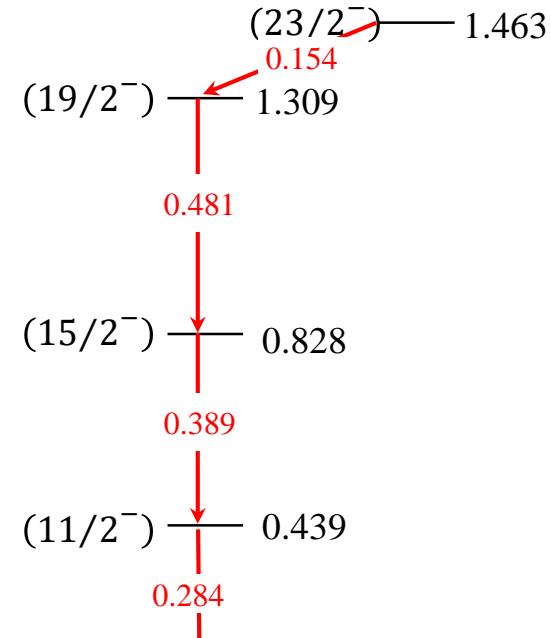
Experimental information on ^{189}Ta

74	W	W155	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190			
73	Ta	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190
72	Hf	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190
71	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	



$9/2^- [514]$

$\alpha = +1/2$ $\alpha = -1/2$



^{189}Ta

gate

481	154		284	389	-
389	154		284	-	481
284	154		-	389	481
343	154	199	246	-	
246	154	199	-	343	370
199	154	-		389	481
154	134	154	246	343	284
134					481

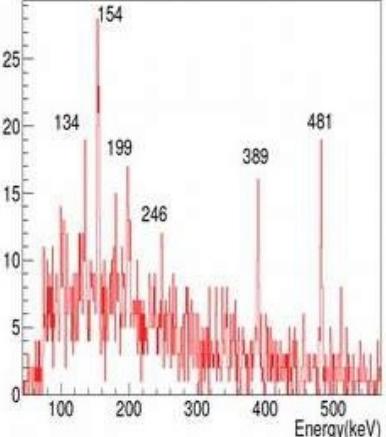
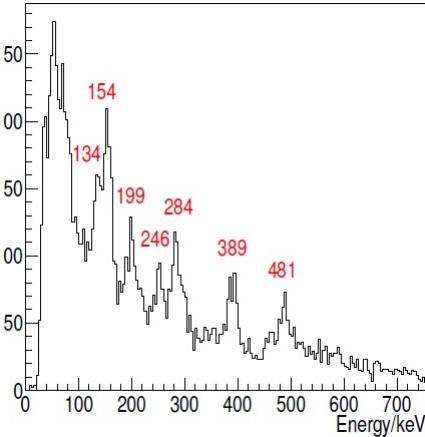
$7/2^+ [404]$

$(9/2^+) \rightarrow 0.154$

$(7/2^+) \rightarrow 0.0$

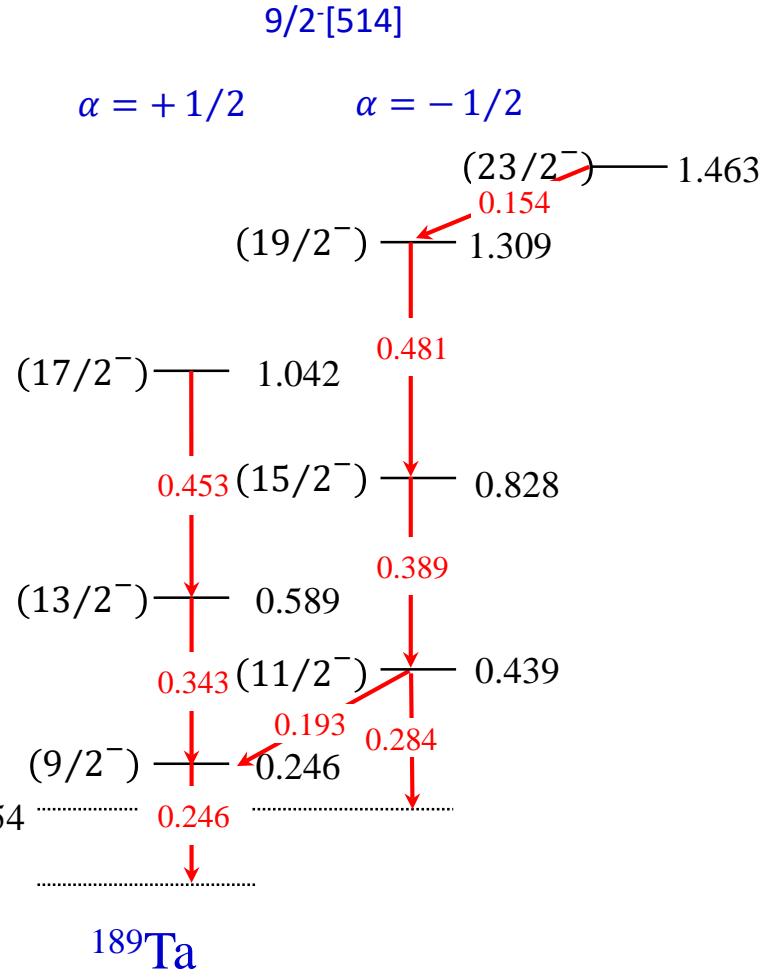
Experimental information on ^{189}Ta

74	W155	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189		
73	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188		
72	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188
71	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188

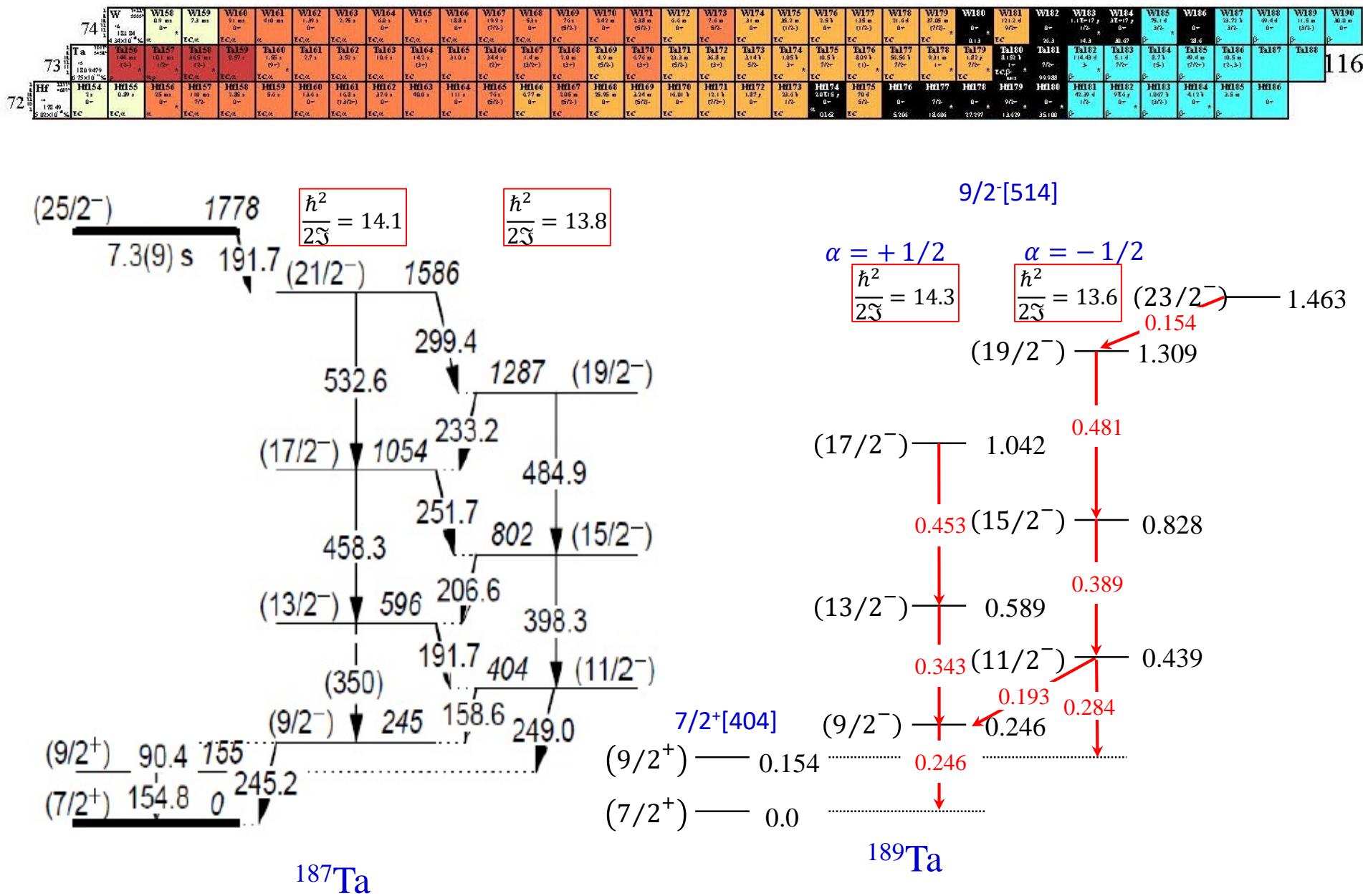


gate

481	154		284	389	-
389	154		284	-	481
284	154		-	389	481
343	154	199	246	-	
246	154	199	-	343	370
199	154	-		389	481
154	134	154	246	343	284
134			343		389



Ta-nuclei: level schemes (1. solution)



Ta-nuclei: level schemes (2. solution)

74	W	W155	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189
73	Ta	54.0	6.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24
72	Hf	54.0	1.04	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	
71	169Lu	54.0	1.04	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	
70	189Ta	54.0	1.04	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	0.22	0.24	

$$\frac{\hbar^2}{2\mathfrak{J}} = 12.6, a = -0.4$$

$$\frac{\hbar^2}{2\mathfrak{J}} = 12.8, a = -0.4$$

$\frac{1}{2}^+[411]$

$\alpha = +1/2$ $\alpha = -1/2$

$$\frac{\hbar^2}{2\mathfrak{J}} = 14.3, a = -0.2 \quad \frac{\hbar^2}{2\mathfrak{J}} = 14.0 \quad (23/2^-) \quad 1.463 \\ 0.154$$

$(19/2^+) \quad 1.309$

0.481

$(17/2^+) \quad 1.042$

0.453 $(15/2^+) \quad 0.828$

0.389

$(13/2^+) \quad 0.589$

0.343 $(11/2^+) \quad 0.439$

0.284

0.193

0.246

0.246

0.246

189Ta

Band(C): $1/2[411]$
 $\alpha=+1/2$ band (1993Og01)

$(17/2^+) \quad 1151.3$

$(13/2^+) \quad 763.1$

$(9/2^+) \quad 449.0$

$(5/2^+) \quad 225.1$

$(1/2^+) \quad 97.4$

$(23/2^+) \quad 1698.2$

$(19/2^+) \quad 1244.0$

$(15/2^+) \quad 842.2$

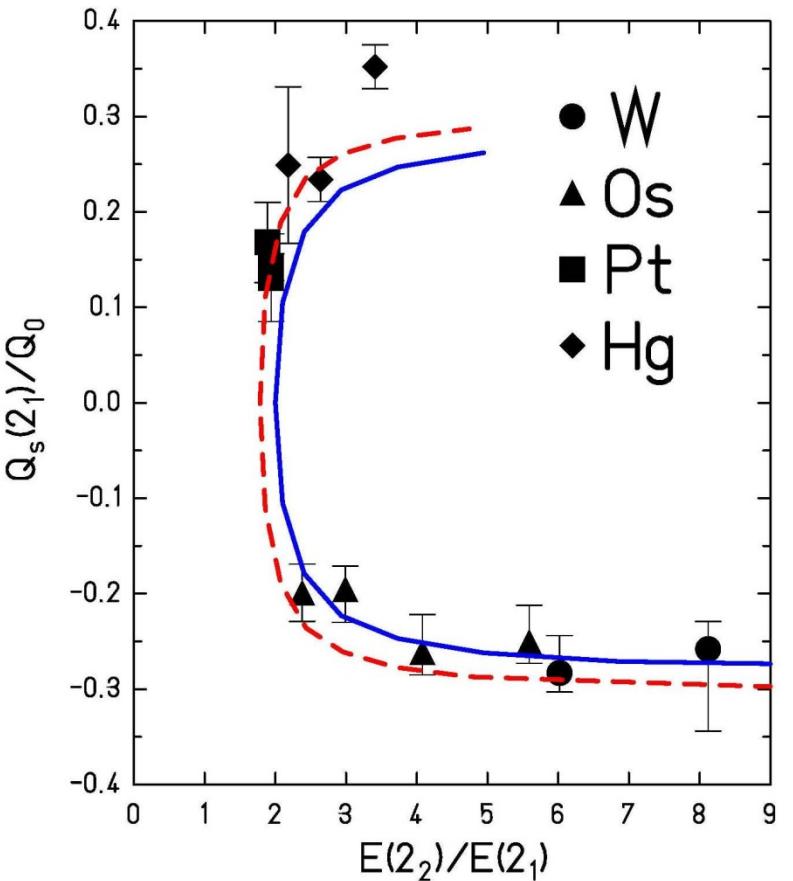
$(11/2^+) \quad 508.4$

$(7/2^+) \quad 260.6$

$(3/2^+) \quad 113.8$

169Lu

Prolate-oblate shape transition critical point N=116



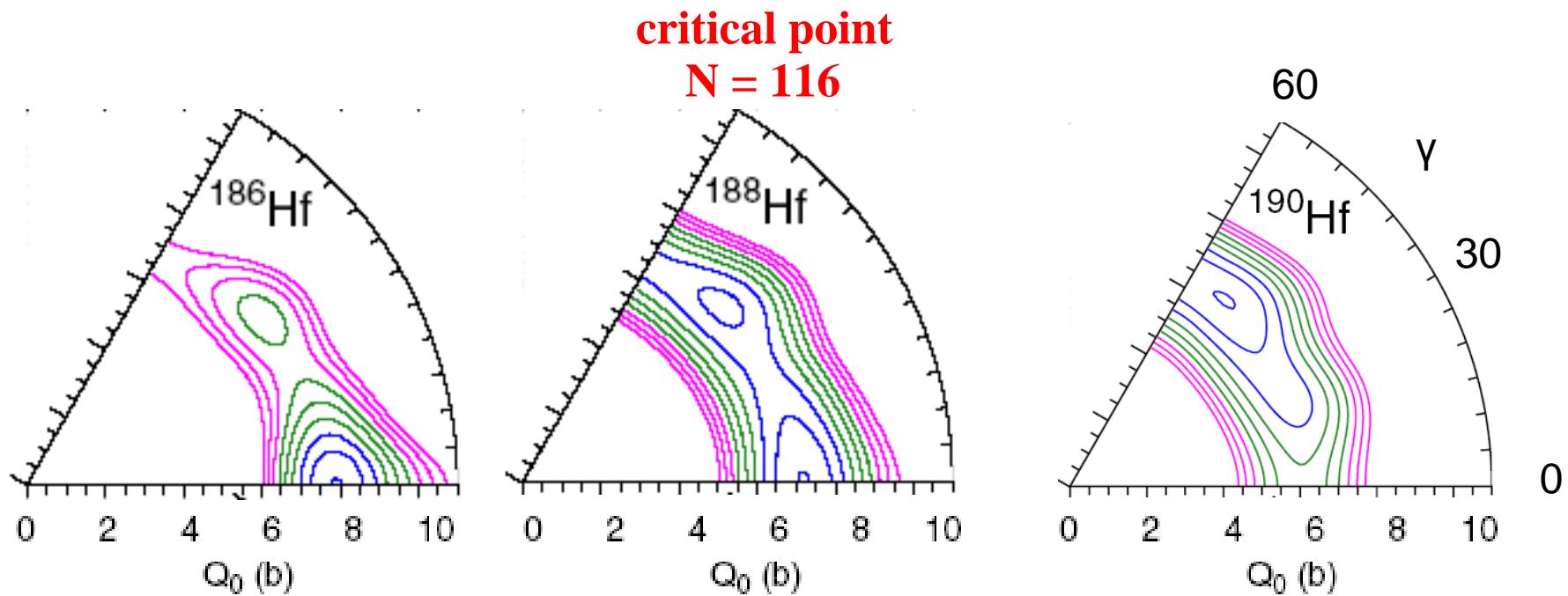
Pb190	Pt191	Pb192	Pb193	Pb194	Pb195	Pb196	Pb197	Pb198	Pb199	Pb200	Pb201	Pb202	Pb203	Pb204	Pb205	Pb206	Pb207	Pb208
1.2 m (3/2) ⁻ EC ₀₊	1.33 m (3/2) ⁻ EC ₀₊	3.2 m (3/2) ⁻ EC ₀₊	11.0 m (1/2) ⁻ EC ₀₊	11.0 m (3/2) ⁻ EC ₀₊	11.0 m (1/2) ⁻ EC ₀₊	11.0 m (3/2) ⁻ EC ₀₊	11.0 m (1/2) ⁻ EC ₀₊	11.0 m (3/2) ⁻ EC ₀₊	1.40 h (1/2) ⁻ EC ₀₊	1.40 h (3/2) ⁻ EC ₀₊	1.40 h (1/2) ⁻ EC ₀₊	1.40 h (3/2) ⁻ EC ₀₊	1.40 h (1/2) ⁻ EC ₀₊	1.40 h (3/2) ⁻ EC ₀₊	1.40 h (1/2) ⁻ EC ₀₊	1.40 h (3/2) ⁻ EC ₀₊	1.40 h (1/2) ⁻ EC ₀₊	
11139	11150	11151	11152	11153	11154	11155	11156	11157	11158	11159	11159	11160	11160	11160	11160	11160	11160	11160
2.6 m (1/2) ⁻ EC ₀₊	9.6 m (1/2) ⁻ EC ₀₊	9.6 m (1/2) ⁻ EC ₀₊	11.6 m (1/2) ⁻ EC ₀₊	11.6 m (1/2) ⁻ EC ₀₊	33.0 m (3/2) ⁻ EC ₀₊	1.54 h (3/2) ⁻ EC ₀₊	1.54 h (1/2) ⁻ EC ₀₊	1.54 h (3/2) ⁻ EC ₀₊	3.78 y 7.41 h EC ₀₊	24.1 h 7.41 h EC ₀₊								
Hg189	Hg189	Hg190	Hg191	Hg192	Hg193	Hg194	Hg195	Hg196	Hg197	Hg198	Hg199	Hg200	Hg201	Hg202	Hg203	Hg204	Hg205	Hg206
8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	8.84 m (3/2) ⁻ EC ₀₊	
Au187	Au188	Au189	Au190	Au191	Au192	Au193	Au194	Au195	Au196	Au197	Au198	Au199	Au200	Au201	Au202	Au203	Au204	Au205
8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	8.4 m (1/2) ⁻ EC ₀₊	
Pr186	Pr187	Pr188	Pr189	Pr190	Pr191	Pr192	Pr193	Pr194	Pr195	Pr196	Pr197	Pr198	Pr199	Pr200	Pr201	Pr202	Pr203	Pr204
1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	1.2 m (3/2) ⁻ EC ₀₊	
Ir185	Ir186	Ir187	Ir188	Ir189	Ir190	Ir191	Ir192	Ir193	Ir194	Ir195	Ir196	Ir197	Ir198	Ir199	Ir200	Ir201	Ir202	Ir203
14.4 h (1/2) ⁻ EC ₀₊	14.64 h (1/2) ⁻ EC ₀₊	10.5 h (1/2) ⁻ EC ₀₊	41.5 h (1/2) ⁻ EC ₀₊	13.4 h (1/2) ⁻ EC ₀₊	11.78 d (1/2) ⁻ EC ₀₊	73.83 d (1/2) ⁻ EC ₀₊	73.83 d (1/2) ⁻ EC ₀₊	1.78 h 3.83 d (1/2) ⁻ EC ₀₊										
Os184	Os185	Os186	Os187	Os188	Os189	Os190	Os191	Os192	Os193	Os194	Os195	Os196	Os197	Os198	Os199	Os200	Os201	Os202
5.623 y 93.6 d 0+	2.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02	1.0023 y 93.6 d 0.02
Re183	Re184	Re185	Re186	Re187	Re188	Re189	Re190	Re191	Re192	Re193	Re194	Re195	Re196	Re197	Re198	Re199	Re200	Re201
37.40 d 0+	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02	37.40 d 0.02
W182	W183	W184	W185	W186	W187	W188	W189	W190	W191	W192	W193	W194	W195	W196	W197	W198	W199	W200
1.11*17 y 37.40 d 0+	1.11*17 y 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02	75.1 d 37.40 d 0.02
26.3	14.2	30.67	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6	28.6

108 110 112 114 116 118 120

isotope	β	γ
¹⁸² W	0.274	11.4 ⁰
¹⁸⁴ W	0.258	13.8 ⁰
¹⁸⁶ W	0.223	15.9 ⁰
¹⁸⁶ Os	0.196	16.5 ⁰
¹⁸⁸ Os	0.185	19.2 ⁰
¹⁹⁰ Os	0.184	22.3 ⁰
¹⁹² Os	0.168	25.2 ⁰
¹⁹² Pt	0.146	-
¹⁹⁴ Pt	0.134	-
¹⁹⁶ Pt	0.135	-
¹⁹⁸ Hg	0.106	36.3 ⁰
²⁰⁰ Hg	0.098	39.1 ⁰
²⁰² Hg	0.082	33.4 ⁰
²⁰⁴ Hg	0.068	31.5 ⁰

Prolate-oblate shape transition

n-rich hafnium ground states



Band crossing prediction in ^{180}Hf

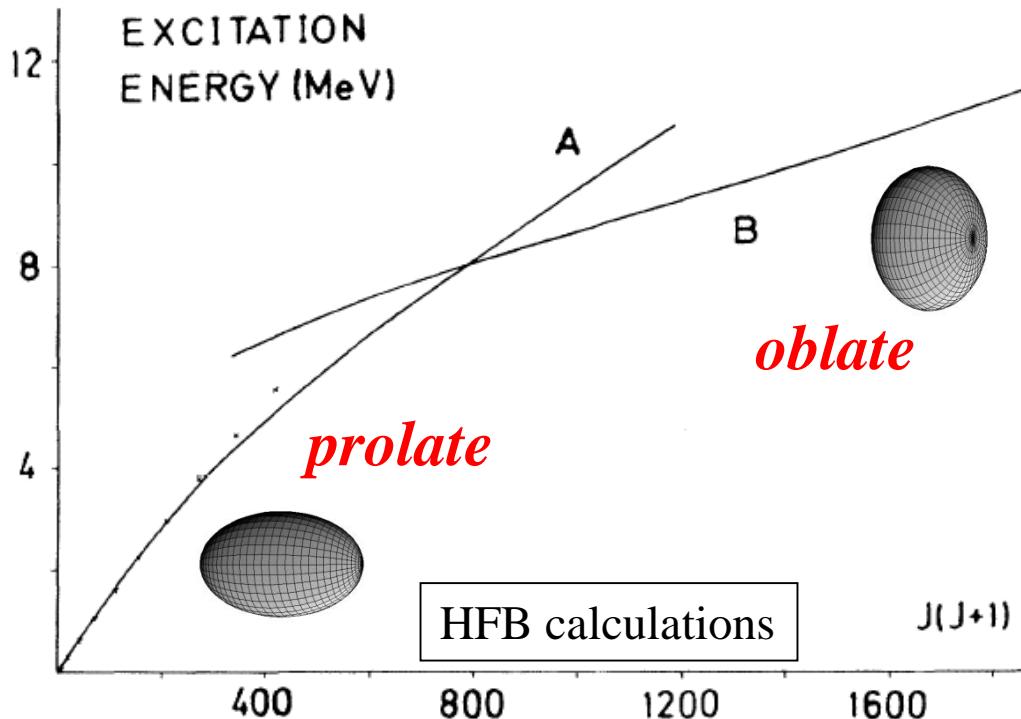
74	W	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190			
73	Ta	144 ap 12.8 120.049 6.25 x 10 ⁻⁴	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188		
72	Hf	144 ap 12.8 120.049 6.25 x 10 ⁻⁴	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188

R. R. Hilton and H. J. Mang

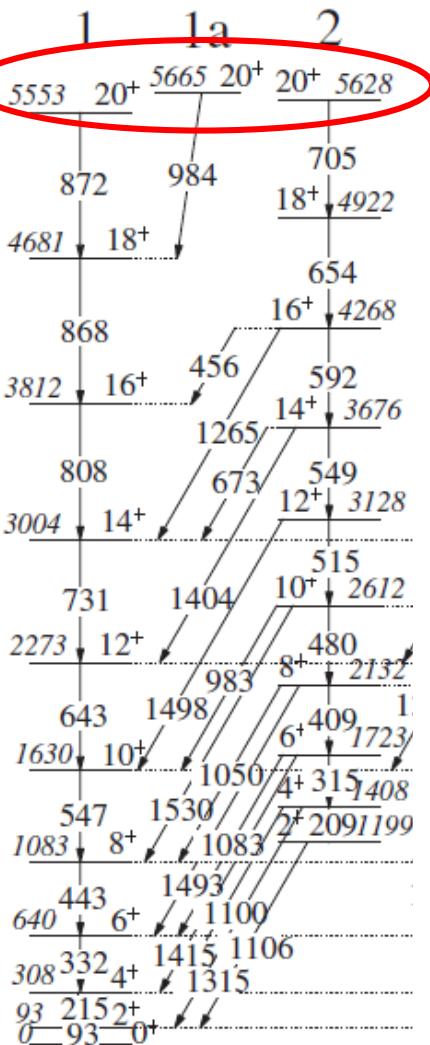
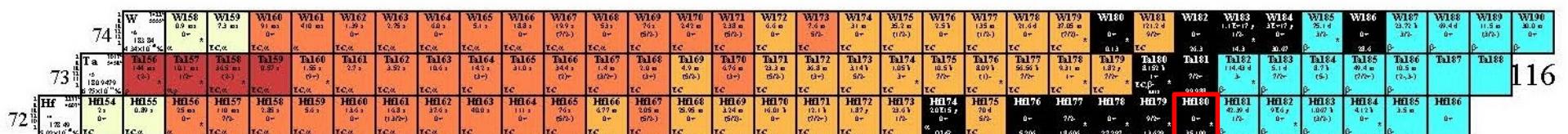
Physik-Department, Technische Universität München, D-8046 Garching, Germany

(Received 6 September 1979)

Giant backbending is predicted to occur in ^{180}Hf at $J \approx 26\hbar$. The effect is clearly seen to be the result of the crossing of two bands with very different intrinsic structure.



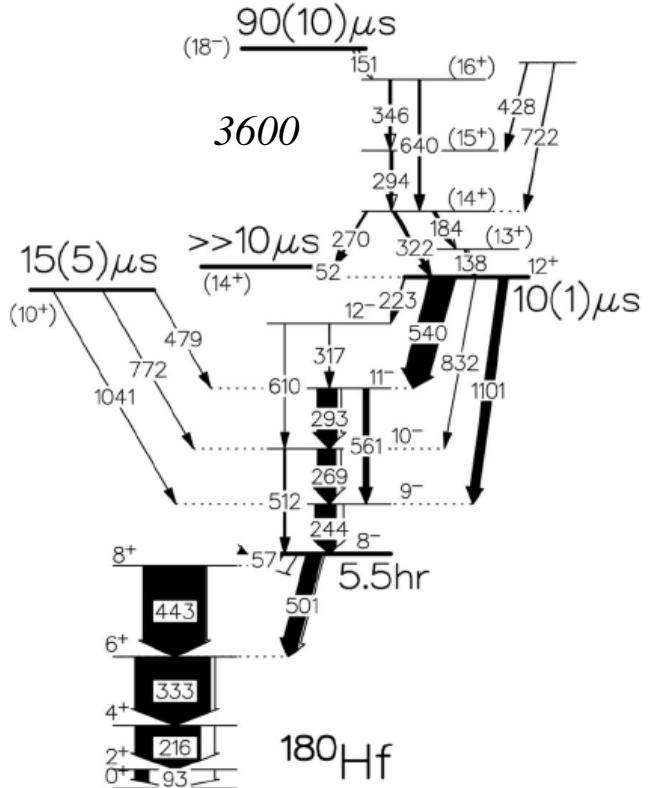
180Hf oblate band?



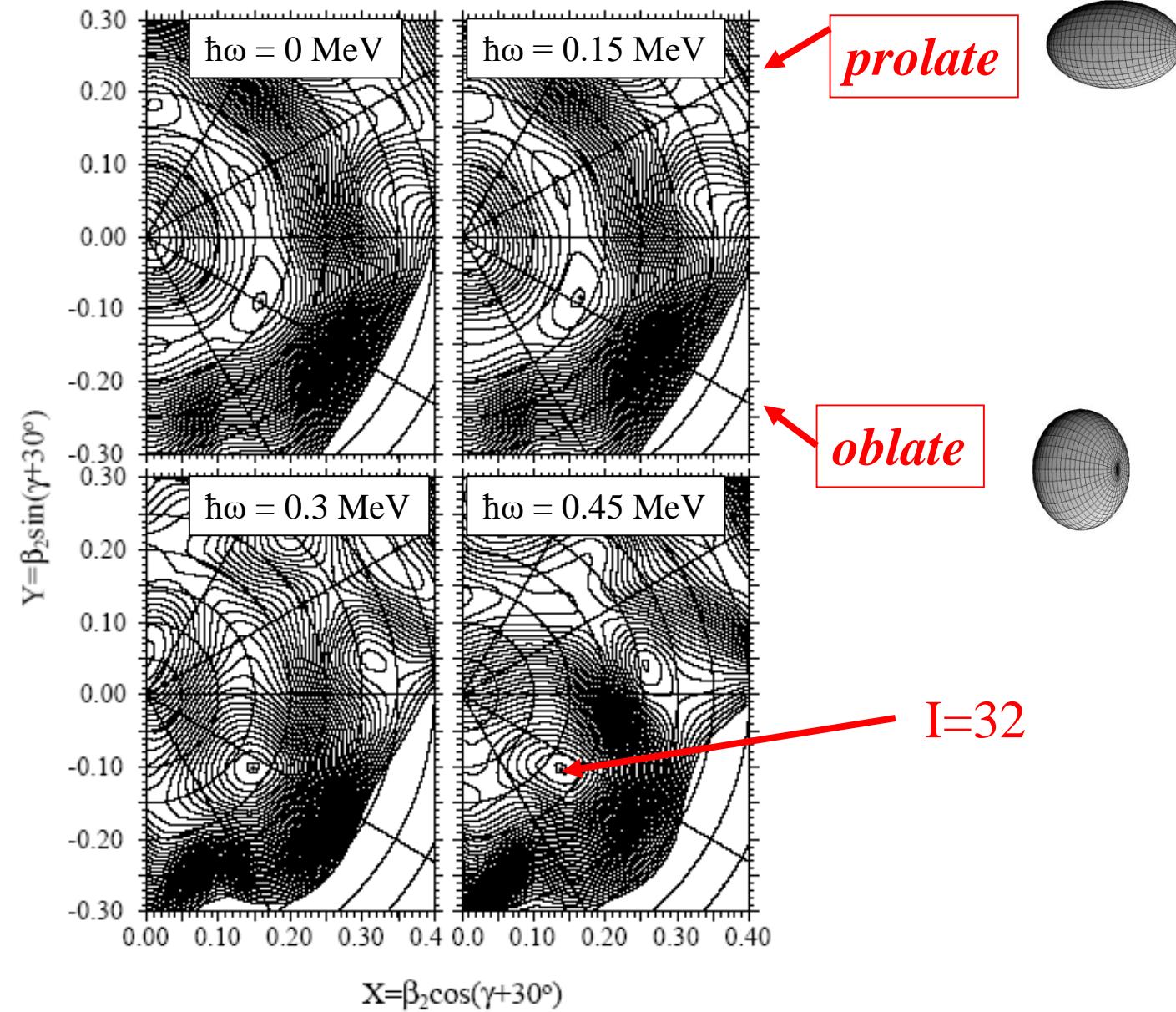
Tandel et al.,
Phys. Rev. Lett. 101 (2008) 182503
with Gammasphere

pre-Gammasphere
high-K yrast isomers:

d'Alarcao et al., Phys. Rev. C59 (1999) 1227(R)

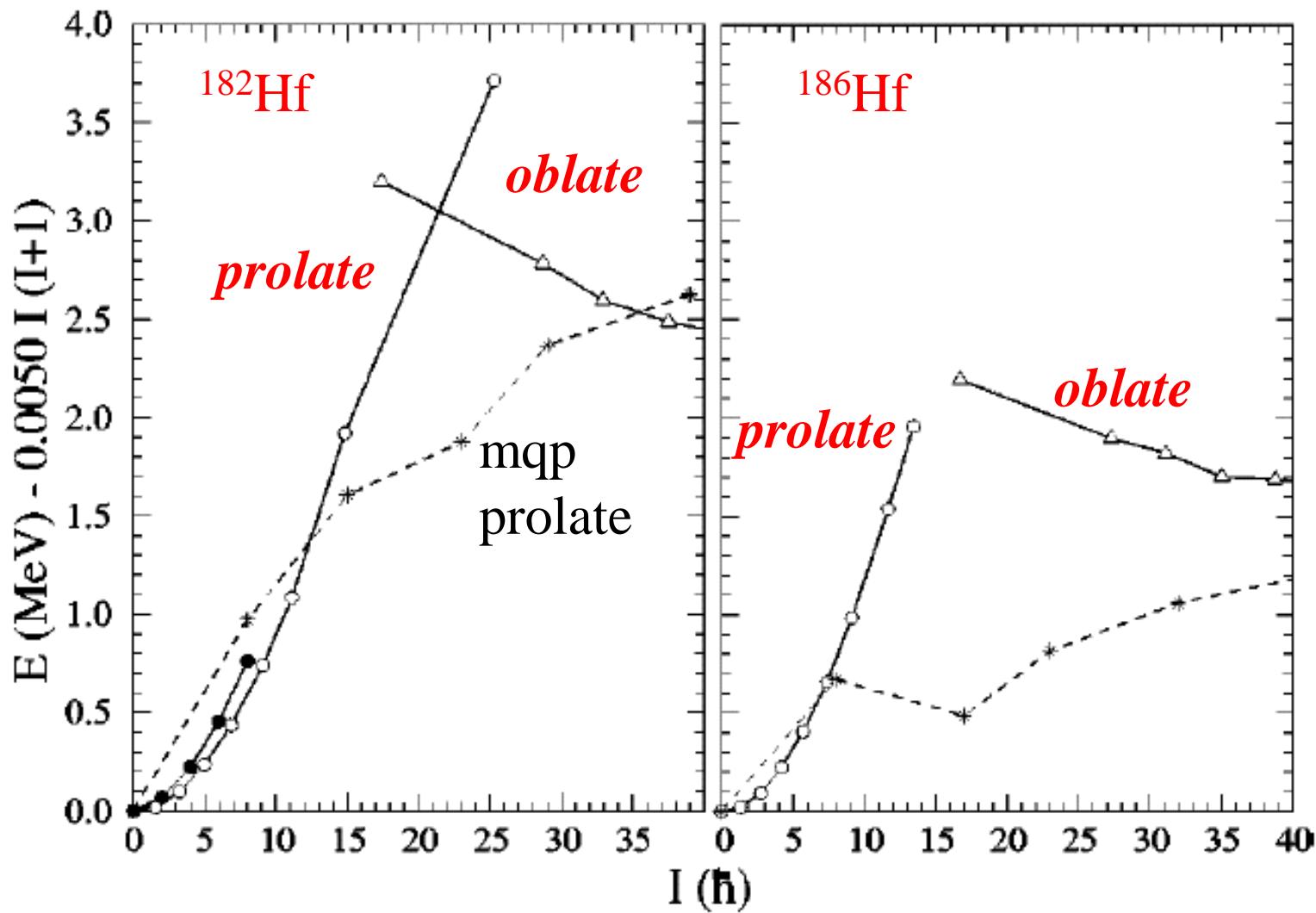


^{190}Hf TRS oblate rotor beyond the critical point



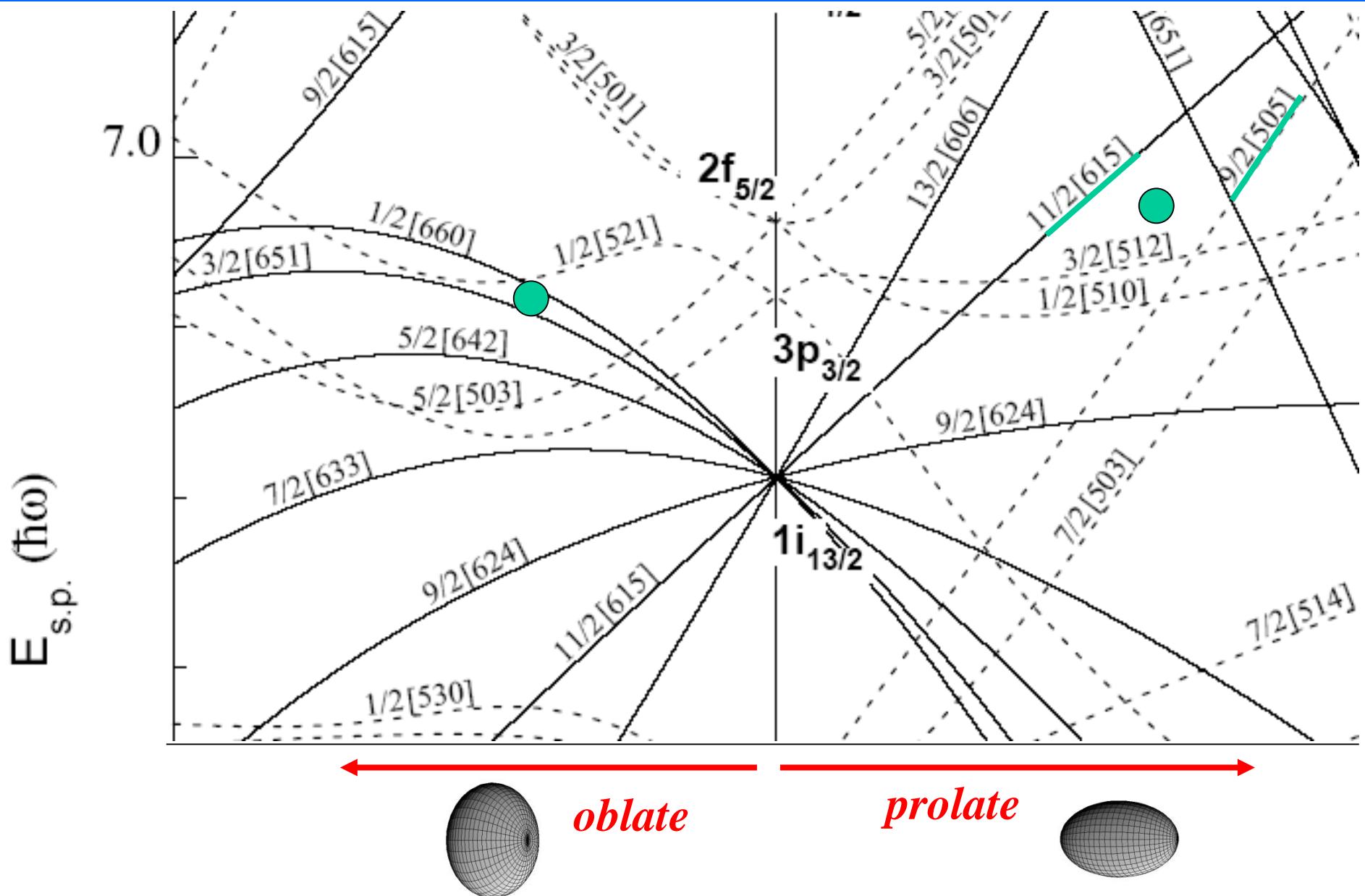
Xu et al., unpublished

Hf prolate vs oblate



Nilsson single-particle diagram

● N = 116 (^{188}Hf , ^{190}W , ^{192}Os)



Appendix: Odd-even nuclei

Er150 18.5 s 0+	Er151 23.5 s (7/2-) \pm	Er152 10.3 s 0+ (7/2-)	Er153 3.75 m 0+ (7/2-)	Er154 5.3 m 0+ (7/2-)	Er155 19.5 m 0+ (7/2-)	Er156 18.65 m 0+ (7/2-)	Er157 2.29 h 3/2- \pm	Er158 2.29 h 0+ 3/2- \pm	Er159 3.6 m 0+ 3/2- \pm	Er160 28.58 h 0+ 3/2- \pm	Er161 3.21 h 0+ 3/2- \pm	Er162 75.0 m 0+ 5/2- \pm	Er163 10.36 h 0+ 5/2- \pm	Er164 10.36 h 0+ 5/2- \pm	Er165 33.6 0+ 7/2+ \pm	Er166 22.95 0+ 7/2+ \pm	Er167 26.8 0+ 7/2+ \pm	Er168 14.9 0+ 7/2+ \pm	Er169 9.40 d 1/2- \pm	Er170 14.9 0+ 7/2+ \pm	Er171 7.516 h 5/2- \pm	Er172 49.3 h 0+ (7/2-)	Er173 1.4 m 0+ (7/2-)	Er174 3.3 m 0+ (9/2+)	Er175 1.2 m (9/2+) \pm
EC	EC	EC , α	EC , α	EC , α	EC , α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC		
Hol149 21.1 s (11/2-) \pm	Hol150 72 s 2- \pm	Hol151 35.2 s 16.8 s (11/2-) \pm	Hol152 2.01 m 2- \pm	Hol153 11.76 m 11/2- \pm	Hol154 48 m 11.76 m (2-) \pm	Hol155 56 m 12.6 m (4+) \pm	Hol156 11.3 m 5/2+ \pm	Hol157 33.05 m 11.3 m 5+ \pm	Hol158 25.6 m 2.48 h 5/2- \pm	Hol159 45.70 y 29 m 5+ \pm	Hol160 25.6 m 2.48 h 5/2- \pm	Hol161 15.0 m 2.48 h 5/2- \pm	Hol162 45.70 y 29 m 1+ \pm	Hol163 29 m 7/2- \pm	Hol164 100 7/2- \pm	Hol165 26.83 h 28.58 h 7/2- \pm	Hol166 26.83 h 28.58 h 0- \pm	Hol167 2.99 m 3.1 h 7/2- \pm	Hol168 2.99 m 3.1 h 0- \pm	Hol169 4.7 m 5/2- \pm	Hol170 2.76 m 53 s (6+) \pm	Hol171 53 s (7/2-)	Hol172 25 s \pm	Hol173 \pm	Hol174 \pm
EC	EC	EC , α	EC , α	EC , α	EC , α	EC , α	EC , α	EC , α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC		
Dy148 3.1 m 0+ (7/2-) \pm	Dy149 4.20 m 0+ (7/2-) \pm	Dy150 7.17 m 0+ 7/2(-)	Dy151 17.9 m 0+ 7/2(-)	Dy152 2.38 h 0+ 7/2(-)	Dy153 6.4 h 0+ 7/2(-)	Dy154 3.0E-6 y 0+ 7/2(-)	Dy155 9.9 h 0+ 7/2(-)	Dy156 0.06 0+ 7/2- \pm	Dy157 0.06 0+ 7/2- \pm	Dy158 0.10 0+ 7/2- \pm	Dy159 144.4 d 0+ 3/2- \pm	Dy160 Dy161 0+ 3/2- \pm	Dy162 0+ 3/2- \pm	Dy163 0+ 5/2- \pm	Dy164 0+ 7/2+ \pm	Dy165 2.334 h 0+ 7/2+ \pm	Dy166 81.6 h 0+ 7/2+ \pm	Dy167 81.6 h 0+ 7/2+ \pm	Dy168 62.0 m 0+ (1/2-)	Dy169 8.7 m 0+ (5/2-)	Dy170 0+ \pm	Dy171 \pm	Dy172 0+ \pm	Dy173 \pm	
EC	EC	EC , α	EC , α	EC , α	EC , α	EC , α	EC , α	EC , α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC		

$$\langle I-2, K | M(E2) | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

$$\langle I_f | M(E2) | I_i \rangle \quad Q_2 \text{ (b)}$$

¹⁶²Dy	2.32 ± 0.02 eb	7.36 ± 0.03
¹⁶³Dy	2.31 ± 0.02 eb	7.29 ± 0.12
¹⁶⁴Dy	2.38 ± 0.01 eb	7.54 ± 0.04
¹⁶⁶Er	2.42 ± 0.01 eb	7.67 ± 0.03
¹⁶⁷Er	2.24 ± 0.01 eb	7.60 ± 0.10
¹⁶⁸Er	2.40 ± 0.02 eb	7.61 ± 0.06

Appendix: Odd-even nuclei

Er150 18.5 s 0+	Er151 23.5 s (7/2-) ±	Er152 10.3 s 0+	Er153 37.1 s (7/2-)	Er154 3.75 m 0+	Er155 5.3 m 7/2-	Er156 19.5 m 0+	Er157 18.65 m 3/2- ±	Er158 2.29 h 0+	Er159 36 m 3/2-	Er160 28.58 h 0+	Er161 3.21 h 3/2-	Er162 0.14 0+	Er163 75.0 m 5/2-	Er164 0.06 0+	Er165 10.36 h 5/2-	Er166 33.6 0+	Er167 22.95 7/2+ ±	Er168 26.8 0+	Er169 9.40 d 1/2-	Er170 14.9 0+	Er171 7.516 h 5/2-	Er172 49.3 h 0+	Er173 1.4 m (7/2-)	Er174 3.3 m 0+	Er175 1.2 m (9/2+)
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	β	β	β	β	β	β	β	β	β
Hol149 21.1 s (11/2-) ±	Hol150 72 s 2- ±	Hol151 35.2 s (11/2-) ±	Hol152 161.8 s 2- ±	Hol153 2.01 m 11/2- ±	Hol154 11.76 m 11/2- ±	Hol155 48 m 48 m	Hol156 56 m 5/2+ ±	Hol157 12.6 s (4+) ±	Hol158 11.3 m 5+ ±	Hol159 33.05 m 7/2- ±	Hol160 25.6 m 5+ ±	Hol161 2.48 h 7/2- ±	Hol162 15.0 m 1+ ±	Hol163 4570 y 7/2- ±	Hol164 29 m 1+ ±	Hol165 100 7/2- ±	Hol166 26.83 h 0- ±	Hol167 2.99 m 7/2- ±	Hol168 4.7 m 3+ ±	Hol169 2.76 m (6+) ±	Hol170 53 s (7/2-)	Hol171 25 s (7/2-)	Hol172 53 s (7/2-)	Hol173 Hol173	Hol174
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2-) ±	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.0E-6 y a	Dy155 9.9 h 3/2- ±	Dy156 3.0E-6 y a	Dy157 0.06 a	Dy158 0.10 EC	Dy159 144.4 d 3/2- ±	Dy160 0+ EC	Dy161 5/2+ 0+	Dy162 0+ 3/2- ±	Dy163 0+ 5/2- ±	Dy164 0+ 2.34	Dy165 2.334 h 7/2+ ±	Dy166 81.6 h 0+ ±	Dy167 620 m (1/2-)	Dy168 8.7 m (5/2-)	Dy169 39 s 0+	Dy170 Dy171	Dy172 0+	Dy173 Dy173	

$$\langle I-2, K | M(E2) | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

$$\langle I-1, K | M(E2) | I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3 \cdot (I+K) \cdot (I-K) \cdot K^2}{(I-1) \cdot I \cdot (I+1)}} \cdot Q_2 e$$

$^{163}\text{Dy}:$ $\frac{B(E2; 5/2 \rightarrow 7/2)}{B(E2; 5/2 \rightarrow 9/2)} = 2.76 \pm 0.14 (2.86_{theo})$

$^{167}\text{Er}:$ $\frac{B(E2; 7/2 \rightarrow 9/2)}{B(E2; 7/2 \rightarrow 11/2)} = 3.81 \pm 0.15 (3.89_{theo})$

Appendix: Odd-even nuclei

Erl50 18.5; 0+	Erl51 23.5; (7/2)-	Erl52 10.3; 0+	Erl53 37.1; (7/2)-	Erl54 3.75 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2- ±	Erl58 2.29 h 0+	Erl59 3.6 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0.14 0+	Erl63 75.0 m 5/2-	Erl64 0+ 5/2-	Erl65 10.36 h 5/2-	Erl66 0+ 33.6	Erl67 7/2+ 22.95	Erl68 0+ 26.8	Erl69 9.40 d 1/2-	Erl70 0+ 14.9	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2)-	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
Hol49 21.1; (11/2)-	Hol50 72; 2- ±	Hol51 35.2; (11/2)-	Hol52 161.8; 2- ±	Hol53 2.01 m 11/2- ±	Hol54 11.76 m 11/2- ±	Hol55 48 m 5/2+	Hol56 12.6 m (4+) ±	Hol57 11.3 m 7/2-	Hol58 33.05 m 5+ ±	Hol59 25.6 m 7/2- ±	Hol60 2.48 h 5+ ±	Hol61 15.0 m 7/2- ±	Hol62 4570 y 1+ ±	Hol63 29 m 7/2- ±	Hol64 1.29 m 1+ ±	Hol65 100 7/2-	Hol66 26.83 h 0- ±	Hol67 2.99 m 3+ ±	Hol68 4.7 m 3.1 h 7/2- ±	Hol69 2.76 m 0- ±	Hol70 53 s (7/2-)	Hol71 25 s (7/2-)	Hol72 53 s (7/2-)	Hol73 Hol73	Hol74 Hol74
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
Dy148 3.1 m 0+	Dy149 4.20 m (7/2)-	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.0E-6 y 0+	Dy155 9.9 h 3/2-	Dy156 0.06 EC	Dy157 0+ EC	Dy158 0.10 EC	Dy159 144.4 d 3/2-	Dy160 0+ EC	Dy161 52+ 2.34	Dy162 0+ 18.9	Dy163 5/2- 25.5	Dy164 0+ 24.9	Dy165 2.334 h 7/2+ ±	Dy166 81.6 h 0+ ±	Dy167 6.20 m 7/2+ ±	Dy168 8.7 m 0+ ±	Dy169 39 s (5/2-)	Dy170 0+	Dy171 Dy171	Dy172 0+	Dy173 Dy173

$^{11/2^-}$ ————— 0.282 $\tau = 0.38 \pm 0.07 \text{ ns}$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

$^{9/2^-}$ ————— 0.167 $\tau = 0.49 \pm 0.09 \text{ ns}$

$^{7/2^-}$ ————— 0.073 $\tau = 2.18 \pm 0.07 \text{ ns}$

$^{5/2^-}$ ————— 0.0

$$\delta_{N \rightarrow M}(\ell) = \left\{ \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} \right\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M | \mathcal{M}(\ell) | I_N \rangle$$

$$\delta_{N \rightarrow M}(E2) = \{1.225 \cdot 10^{13} \cdot E_{\gamma}^5 (\text{MeV})^5\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M | \mathcal{M}(E2) | I_N \rangle$$

$$\delta_{N \rightarrow M}(M1) = \{1.758 \cdot 10^{13} \cdot E_{\gamma}^3 (\text{MeV})^3\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M | \mathcal{M}(M1) | I_N \rangle$$

$$\varepsilon_{N \rightarrow M}^2(\ell) = \delta_{N \rightarrow M}^2(\ell) \cdot \alpha_{N \rightarrow M}(\ell)$$

conversion coefficient.: bricc.anu.edu.au

Appendix: Odd-even nuclei

Erl50 18.5; 0+	Erl51 23.5; (7/2)-	Erl52 10.3; 0+	Erl53 37.1; (7/2)-	Erl54 3.75 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2- ±	Erl58 2.29 h 0+	Erl59 3.6 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0.14 EC	Erl63 75.0 m 5/2-	Erl64 0+ EC	Erl65 10.36 h 5/2-	Erl66 0+ EC	Erl67 7/2+ ±	Erl68 0+ EC	Erl69 9.40 d 1/2-	Erl70 0+ EC	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2)-	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)	
EC	EC	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC	EC	EC	EC	EC	EC	EC	EC	EC	33.6	22.95	26.8	β^-	14.9	β^-	β^-	β^-	β^-	β^-	β^-	
Hol149 21.1; (11/2)- ±	Hol150 72; 2- ±	Hol151 35.2; 16.8; ±	Hol152 2.01 m 2- ±	Hol153 11.76 m 11/2- ±	Hol154 48 m 11/2- ±	Hol155 56 m 5/2+ ±	Hol156 12.6 m (4+) ±	Hol157 11.3 m 7/2- ±	Hol158 33.05 m 5+ ±	Hol159 25.6 m 7/2- ±	Hol160 2.48 h 5+ ±	Hol161 15.0 m 7/2- ±	Hol162 4570 y 1+ ±	Hol163 29 m 7/2- ±	Hol164 1+ EC, <i>b</i>	Hol165 100	Hol166 26.83 h β^-	Hol167 2.99 m 3+ ±	Hol168 3.1 h 7/2- ±	Hol169 4.7 m 7/2- ±	Hol170 2.76 m (6+) ±	Hol171 53 s (7/2-)	Hol172 25 s	Hol173 Hol174	Hol174	
EC	EC	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC	EC	EC	EC	EC	EC	EC	EC	EC	β^-	β^-	β^-	β^-	β^-	β^-	β^-	β^-	β^-	β^-		
Dy148 3.1 m 0+	Dy149 4.20 m (7/2)- ±	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.0E-6 y a	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 3/2- ±	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 52+ EC	Dy162 0+	Dy163 5/2- ±	Dy164 0+	Dy165 2.334 h 7/2+ ±	Dy166 81.6 h 0+	Dy167 2.334 h 7/2+ ±	Dy168 81.6 h 0+	Dy169 39 s (5/2-)	Dy170 0+	Dy171 Dy172	Dy173 Dy173		
EC	EC	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC, <i>a</i>	EC	EC	EC	EC	EC	EC	EC	EC	EC	2.34	18.9	25.5	24.9	28.2	β^-	β^-	β^-	β^-	β^-	β^-	β^-

$^{11/2} \text{--}$ 0.282

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

$^{9/2} \text{--}$ 0.167 $\tau = 0.49 \pm 0.09 \text{ ns}$

$^{7/2} \text{--}$ 0.073 $\tau = 2.18 \pm 0.07 \text{ ns}$

$^{5/2} \text{--}$ 0.0

^{163}Dy

Appendix: Odd-even nuclei

Erl50 18.5; 0+	Erl51 23.5; (7/2)-	Erl52 10.3; 0+	Erl53 37.1; (7/2)-	Erl54 3.75 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2- ±	Erl58 2.29 h 0+	Erl59 3.6 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0.14 0+	Erl63 75.0 m 5/2-	Erl64 1.61 0+	Erl65 10.36 h 5/2-	Erl66 33.6 0+	Erl67 22.95 7/2+ ±	Erl68 26.8 0+	Erl69 9.40 d 1/2-	Erl70 14.9 0+	Erl71 7.516 h 0+	Erl72 49.3 h 0+	Erl73 1.4 m (7/2)-	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
Hol49 21.1; (11/2)-	Hol50 72; 2- ±	Hol51 35.2; (11/2)-	Hol52 161.8; 2- ±	Hol53 2.01 m 11/2- ±	Hol54 11.76 m 11/2- ±	Hol55 48 m 5/2+	Hol56 12.6 m (4+) ±	Hol57 56 m 7/2-	Hol58 11.3 m 5+ ±	Hol59 33.05 m 7/2- ±	Hol60 25.6 h 5+ ±	Hol61 2.48 h 7/2- ±	Hol62 15.0 m 7/2- ±	Hol63 4570 y 7/2- ±	Hol64 29 m 1+ ±	Hol65 100 7/2-	Hol66 26.83 h 0- ±	Hol67 2.99 m 7/2- ±	Hol68 3.1 h 3+ ±	Hol69 4.7 m 7/2- ±	Hol70 2.76 m (6+) ±	Hol71 53 s (7/2-)	Hol72 25 s 0+	Hol73 Hol73	Hol74
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
Dy148 3.1 m 0+	Dy149 4.20 m (7/2)-	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.0E-6 y 0+	Dy155 9.9 h 3/2-	Dy156 Dy156 0.06	Dy157 3/2- ±	Dy158 Dy158 0.10	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 52+ 0+	Dy162 0+	Dy163 5/2- ±	Dy164 0+	Dy165 2.334 h 7/2+ ±	Dy166 81.6 h 0+	Dy167 81.6 h 7/2+ ±	Dy168 6.20 m (1/2-)	Dy169 8.7 m (5/2-)	Dy170 39 s 0+	Dy171 Dy171	Dy172 0+	Dy173
EC	EC	EC	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC

$^{13/2} \text{——}$ 0.295 $\tau = 0.38 \pm 0.07 \text{ ns}$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

$^{11/2} \text{——}$ 0.178 $\tau = 0.49 \pm 0.09 \text{ ns}$

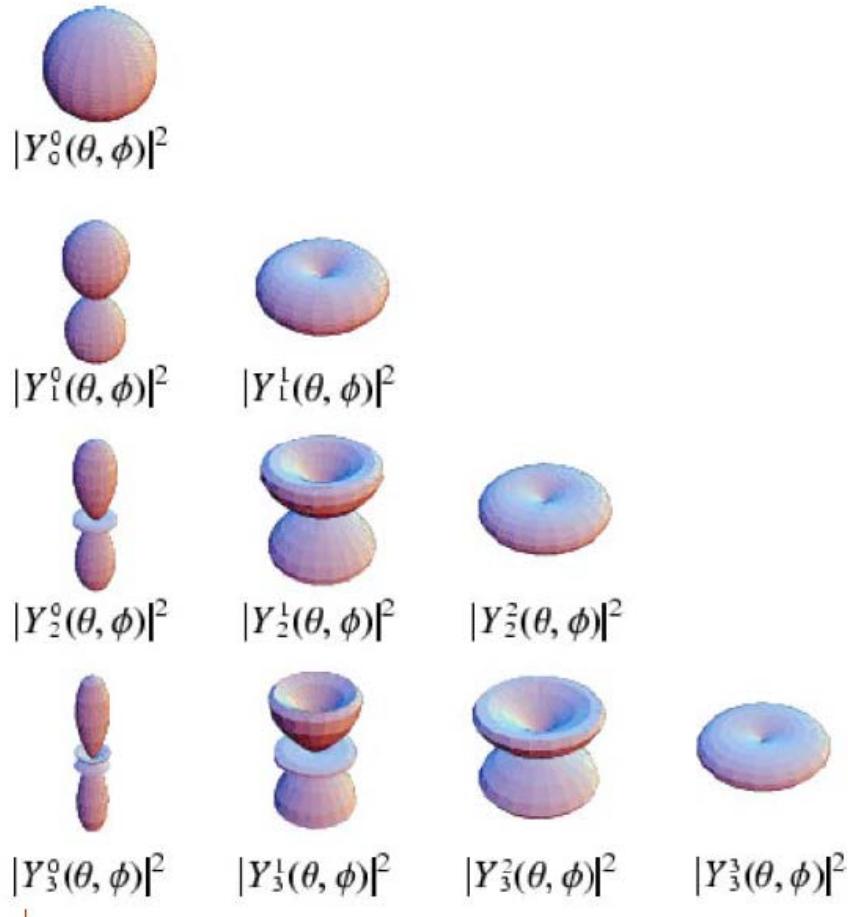
$^{9/2} \text{——}$ 0.079 $\tau = 2.18 \pm 0.07 \text{ ns}$

$^{7/2} \text{——}$ 0.0

^{167}Er

Spin I	E_{γ} (MeV)	$(2I+1)^{-1/2}$	$\langle I-1//M()//I\rangle$	delta	α_T	ε^2	τ (ns)
7/2	0.0734	0.3536	-3.886 (E2)	-7019.1	8.9	$4.38 \cdot 10^8$	2.05
9/2	0.0939	0.3162	0.108 (M1)	3183.7	5.7	$5.78 \cdot 10^7$	1.80
	0.167	0.3162	-4.002 (E2)	-11968	3.4	$4.87 \cdot 10^8$	1.59
			0.153 (M1)	5837.1	2.9	$9.88 \cdot 10^7$	1.31
		2.299 (E2)	29133	0.4	$3.65 \cdot 10^8$	0.51	

Appendix: Spherical harmonics



$$Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta, \phi) = \frac{1}{2} \cdot \sqrt{\frac{3}{\pi}} \cdot \cos \theta$$

$$Y_{1\pm 1}(\theta, \phi) = \mp \frac{1}{2} \cdot \sqrt{\frac{3}{2\pi}} \cdot \sin \theta \cdot e^{\pm i\phi}$$

$$Y_{20}(\theta, \phi) = \sqrt{\frac{5}{16\pi}} \cdot (3 \cdot \cos^2 \theta - 1)$$

$$Y_{2\pm 1}(\theta, \phi) = \mp \sqrt{\frac{15}{8\pi}} \cdot \sin \theta \cdot \cos \theta \cdot e^{\pm i\phi}$$

$$Y_{2\pm 2}(\theta, \phi) = \sqrt{\frac{15}{32\pi}} \cdot \sin^2 \theta \cdot e^{\pm 2i\phi}$$

$$Y_{30}(\theta, \phi) = \sqrt{\frac{7}{16\pi}} \cdot (2 \cos^3 \theta - 3 \cos \theta \sin^2 \theta)$$

$$Y_{3\pm 1}(\theta, \phi) = \mp \sqrt{\frac{21}{64\pi}} \cdot (4 \cos^2 \theta \sin \theta - \sin^3 \theta) \cdot e^{\pm i\phi}$$

$$Y_{3\pm 2}(\theta, \phi) = \sqrt{\frac{105}{32\pi}} \cdot \cos \theta \sin^2 \theta \cdot e^{(\pm 2)i\phi}$$

$$Y_{3\pm 3}(\theta, \phi) = \mp \sqrt{\frac{35}{64\pi}} \cdot \sin^3 \theta \cdot e^{(\pm 3)i\phi}$$