Outline: spin and parity

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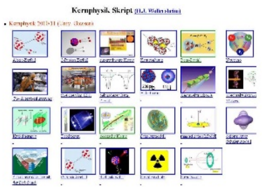
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. basics
2. angular correlation and distribution
3. linear polarization

Spins and parities

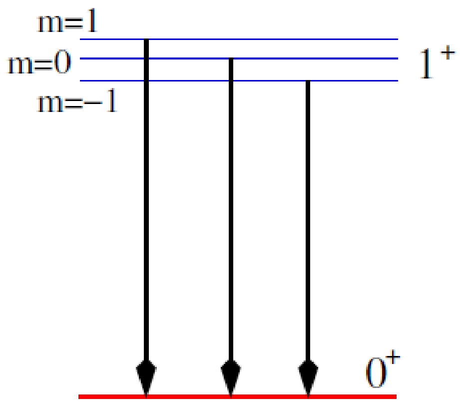
Two distinct types of measurements:

Angular correlation : can be done with a non-aligned source but need γ-γ coincidence information.

Angular distribution: need an aligned source but can be done with singles data.

...note that these cannot measure parity but you can usually infer something about the transition

The basics of the situation



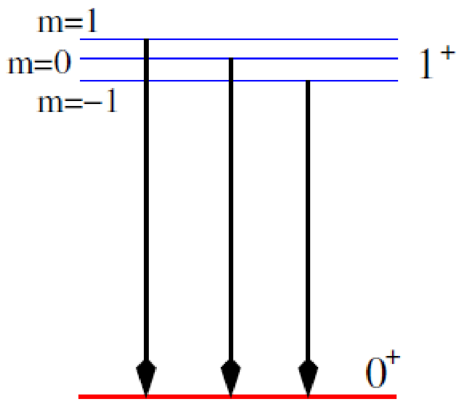
Imagine the situation of an M1 decay between two states, the initial one has J^{π} value of 1^{+} and the final one a J^{π} of 0^{+}

The initial $J^{\pi}=1^{+}$ state has 3 degenerate magnetic substates which differ by the magnetic quantum numbers m of ± 1 and 0.

The final $J^{\pi}=0^{+}$ state has a single magnetic substate with m=0.

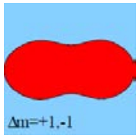
When the substates of $J^{\pi}=1^{+}$ state decay, the γ-rays emitted have different angular patterns.

The basics of the situation

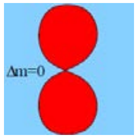


For the M1 case the angular distributions W(θ) are:

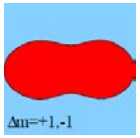
$$W_{M1,\Delta m=1}(\theta) = \frac{3}{16\pi}(1+cos^2\theta)$$



$$W_{M1,\Delta m=0}(\theta) = \frac{3}{8\pi}sin^2\theta$$

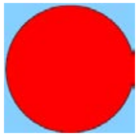


$$W_{M1,\Delta m=-1}(\theta) = \frac{3}{16\pi}(1+cos^2\theta)$$



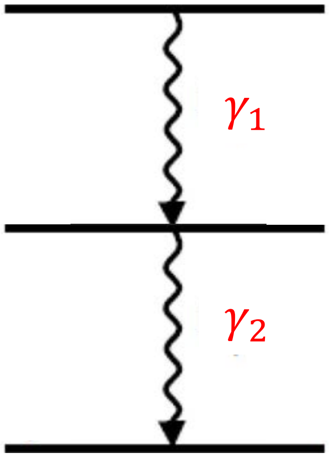
So the total distribution is $W_{M1} = \frac{1}{3}W_{M1,\Delta m=1} + \frac{1}{3}W_{M1,\Delta m=0} + \frac{1}{3}W_{M1,\Delta m=-1}$

$$= \frac{1}{8\pi}(1+cos^2\theta+sin^2\theta) = \frac{1}{4\pi}$$



no angular dependence

Angular correlation – non-oriented source

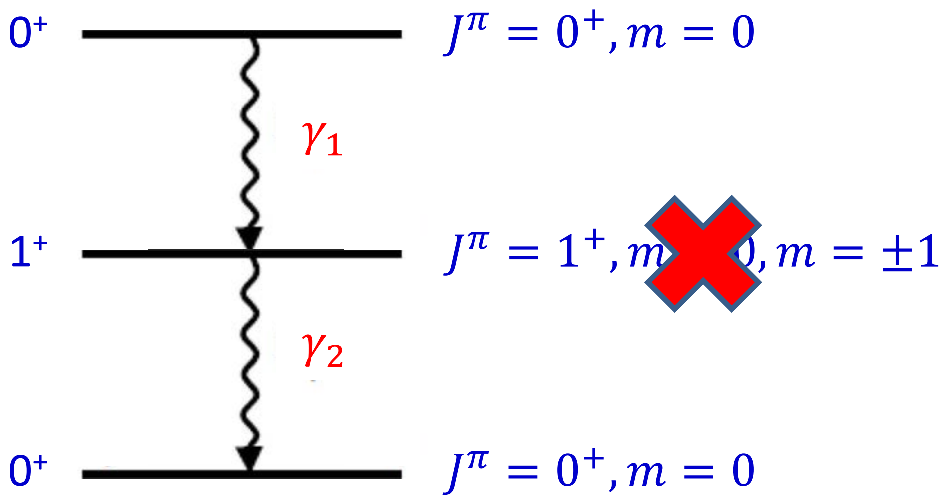


Let's imagine we have two γ-rays which follow immediately after each other in the level scheme.

If we measure γ_1 or γ_2 in singles, then the distribution will be isotropic (same intensity at all angles) ... there is no preferred direction of emission

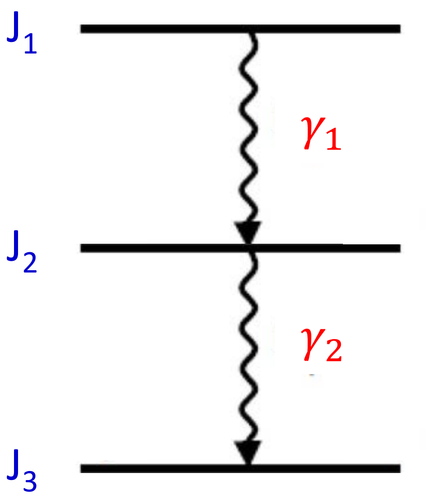
Now imagine that we measure γ_1 and γ_2 in coincidence. We say that measuring γ_1 causes the intermediate state to be aligned. We define the z-direction as the direction of γ_1

The angular distribution of the emission of γ_2 then depends on the spin/parities of the states involved and on the multipolarity of the transition.



Hence, for γ_2 we only see the m = ±1 to m = 0 part of the distribution i.e. we see that the intensity measured as a function of angle (relative to γ_1) follows a $1 + cos^2\theta$ distribution.

General formula



In general, the γ-ray intensity varies as:

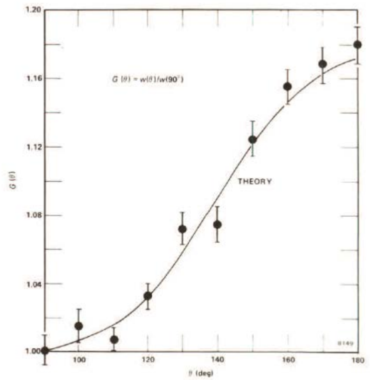
$$W(\theta) = \sum_{k_{even}} A_k(\gamma_1)A_k(\gamma_2)Q_k(\gamma_1)Q_k(\gamma_2)P_k(cos\theta)$$

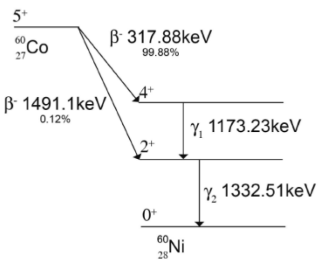
where
θ is the relative angle between the two γ-rays
Q_k accounts for the fact that we do not have point detectors
A_k depends on the details of the transition and the spins of the level

$$P_0 = 1 \quad P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1) \quad P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$$

$$W(\theta) = 1 + a_2cos^2\theta + a_4cos^4\theta$$

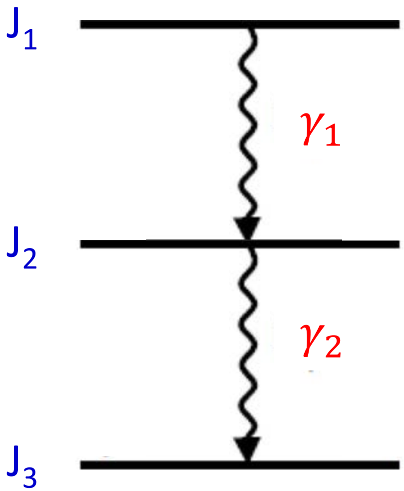
I_1 (ℓ_1)	I_2 (ℓ_2)	I_3	a_2	a_4
0 (1)	1 (1)	0	1	0
1 (1)	1 (1)	0	-1/3	0
1 (2)	1 (1)	0	-1/3	0
2 (1)	1 (1)	0	1/13	0
3 (2)	1 (1)	0	-3/29	0
0 (2)	2 (2)	0	-3	4
1 (1)	2 (2)	0	-1/3	0
2 (1)	2 (2)	0	3/7	0
2 (2)	2 (2)	0	-15/13	16/13
3 (2)	2 (2)	0	-3/29	0
4 (2)	2 (2)	0	1/8	1/24





R.D. Evans, The Atomic Nucleus

General formula



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$$A_k(\gamma_1) = \frac{F_k(J_2 J_1 \ell, \ell) - 2 \cdot \delta \cdot F_k\,(J_2 J_1 \ell, \ell + 1) + \delta^2 \cdot F_k\,(J_2 J_1 \ell + 1, \ell + 1)}{1 + \delta^2}$$

$$A_k(\gamma_2) = \frac{F_k(J_2 J_3 L, L) - 2 \cdot \delta \cdot F_k\,(J_2 J_3 L, L + 1) + \delta^2 \cdot F_k\,(J_2 J_3 L + 1, L + 1)}{1 + \delta^2}$$

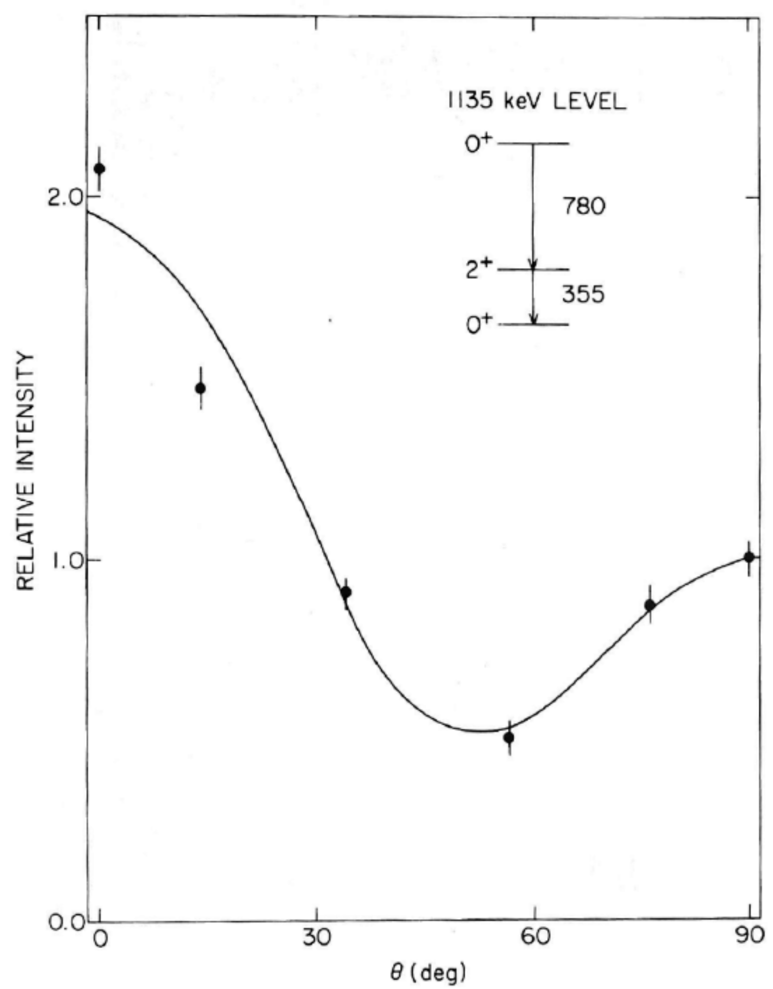
Ferentz-Rosenzweig coefficients

$$F_k(LL'I_1 I_2) = (-1)^{I_1 + I_2 + 1} \sqrt{2k+1} \sqrt{2L+1} \sqrt{2L'+1} \sqrt{2I_2+1} \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I_1 & I_1 & I_2 \end{Bmatrix}$$

https://griffincollaboration.github.io/AngularCorrelationUtility/

A special case:

$^{195}_{78}Pt(n,\gamma)^{196}_{78}Pt$



Angular correlations with arrays

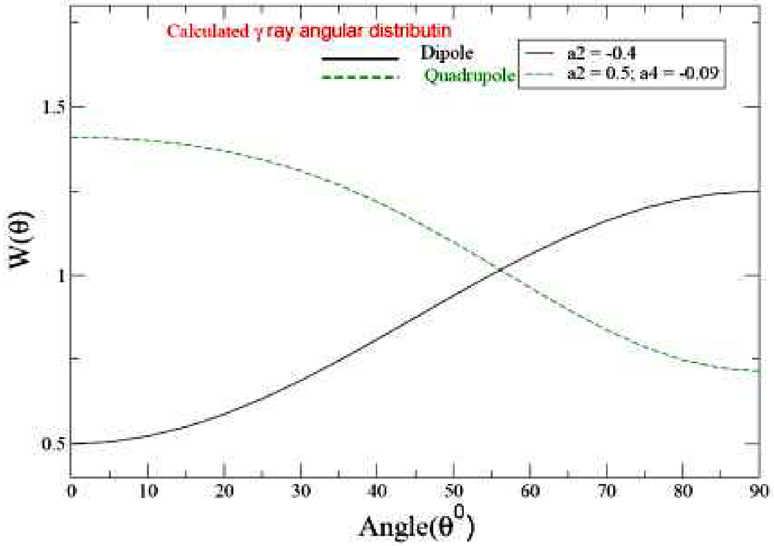
Many arrays are designed symmetrically, so the range of possible angles is reduced.

Therefore one measures a Directional Correlation from Oriented Nuclei (DCO ratio)

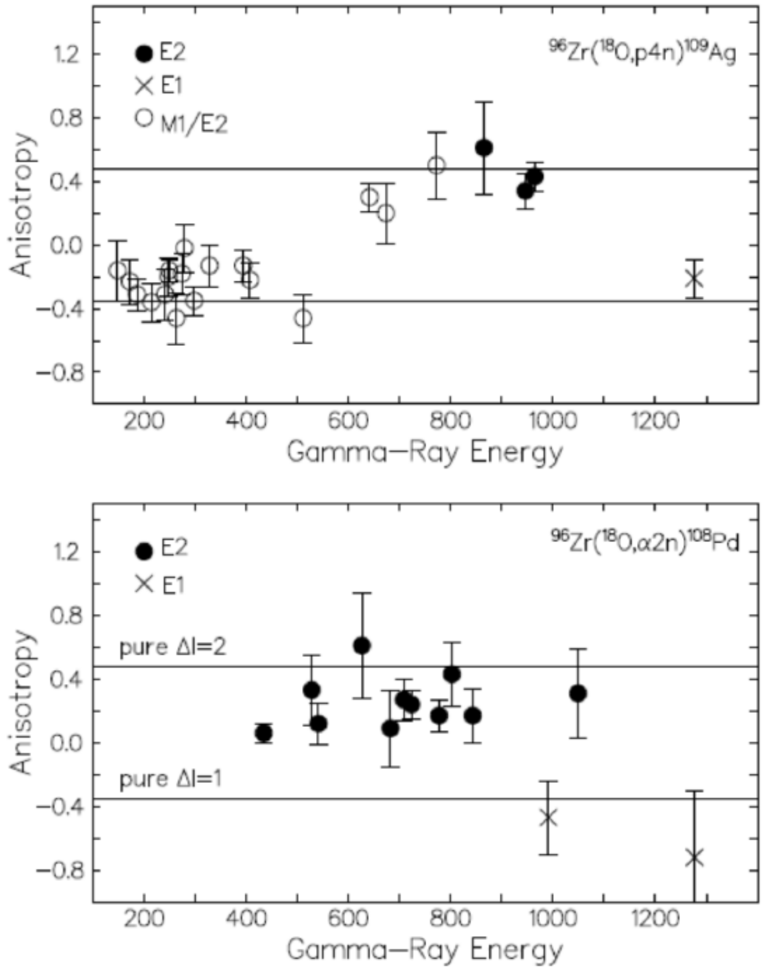
In the simplest case, if you have an array with detectors at 35^0 and 90^0.
Gate on 90^0 detector, measure coincident intensities in

- other 90^0 detectors
- 35^0 detectors

Take the ratio and compare with calculations ... can usually separate quadrupoles from dipoles but cannot measure mixing ratios



Angular correlations with arrays

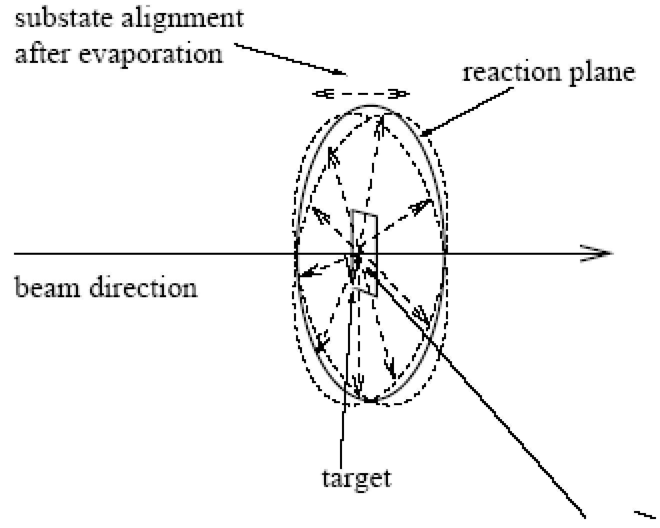


K.R.Pohl et al., Phys Rev C53 (1996) 2682

Angular distribution

In heavy-ion fusion-evaporation reactions, the compound nuclei have their spin aligned in a plane perpendicular to the beam axis:

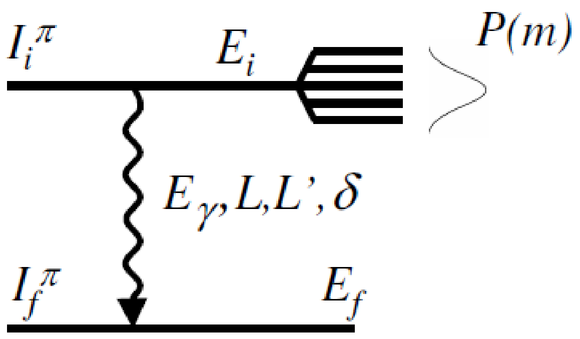
$$\vec{\ell} = \vec{r} \times \vec{p}$$



Depending on the number and type of particles 'boiled off' before a γ-ray is emitted, transitions are emitted from oriented nuclei and therefore their intensity shows an angular dependence.

$$W(\theta) = A_0\left(1 + \frac{A_2}{A_0} \cdot B_2 \cdot Q_2 \cdot P_2\,(cos\theta) + \frac{A_4}{A_0} \cdot B_4 \cdot Q_4 \cdot P_4(cos\theta)\right)$$

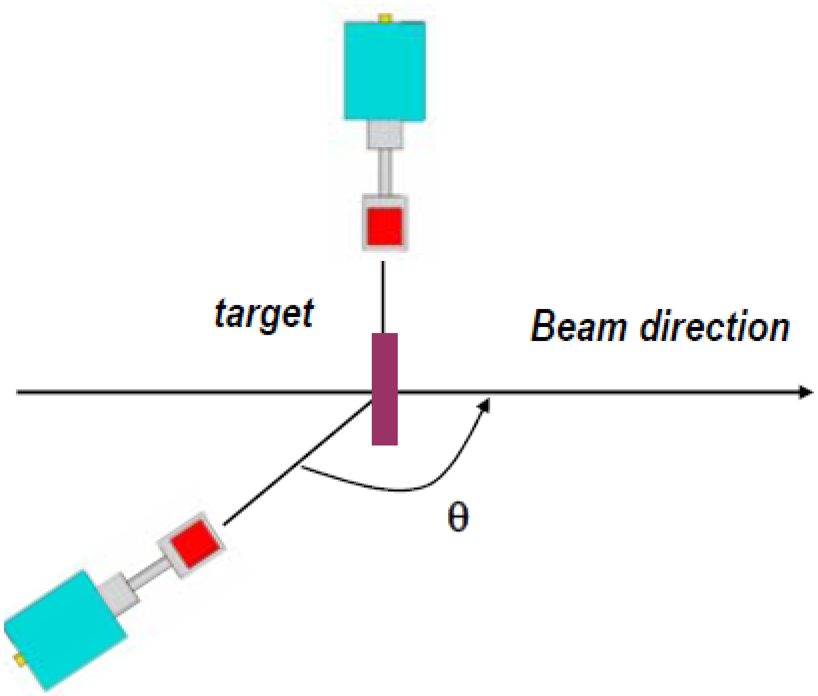
where A_k, Q_k and P_k are as before and B_k contains information about the alignment of the state

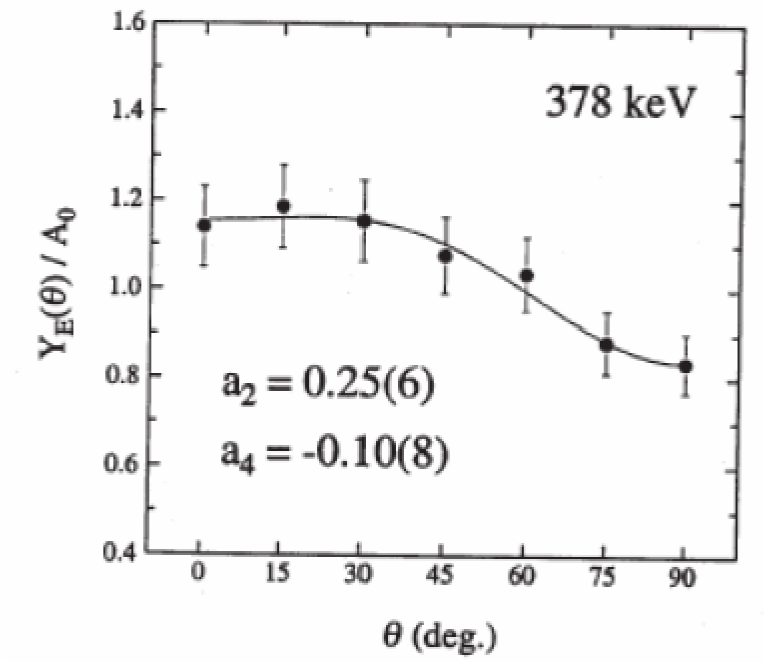


$$B_k(I_i) = \sqrt{2I_i + 1} \sum_{m=-I}^{+I} (-1)^{I_i - m} \langle I_i m I_i - m | k0 \rangle P(m)$$

$$P(m) = \frac{exp\left(-\frac{m^2}{2\sigma^2}\right)}{\sum_{m'=-I}^{+I} exp\left(-\frac{m'^2}{2\sigma^2}\right)}$$

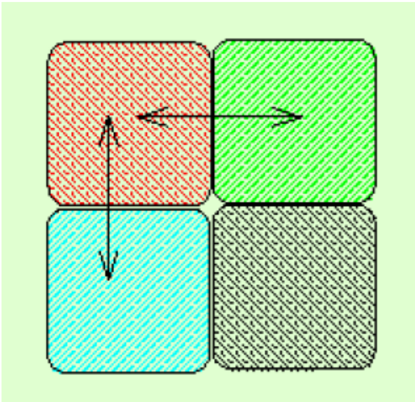
Angular distribution



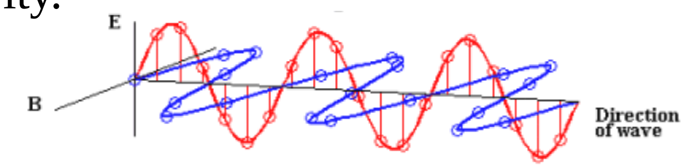


Measure: the γ-ray yield as a function of θ

Linear polarization



A segmented detector can be used to measure the linear polarization which can be used to distinguish between magnetic (M) and electric (E) character of radiation of the same multipolarity.



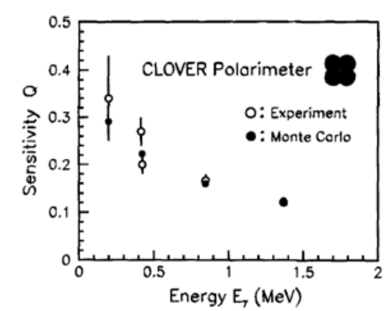
The Compton scattering cross section is larger in the direction perpendicular to the electrical field vector of the radiation.

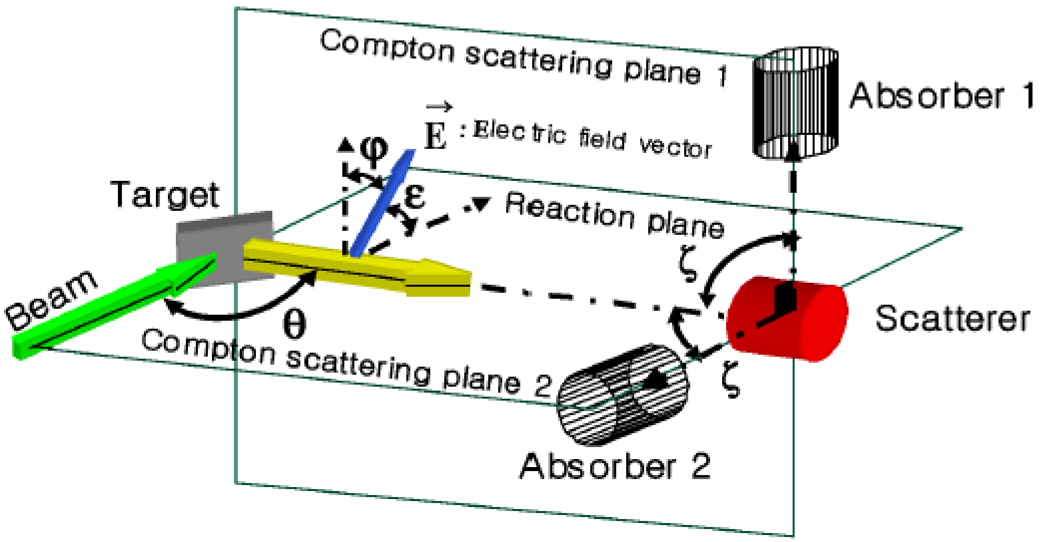
Define experimental asymmetry as: $A = \frac{N_{90} - N_0}{N_{90} + N_0}$

where N_{90} and N_0 are the intensities of scattered photons perpendicular and parallel to the reaction plane.

The experimental linear polarization P=A/Q where Q is the polarization sensitivity of the detector

Q ~ 13% at 1 MeV



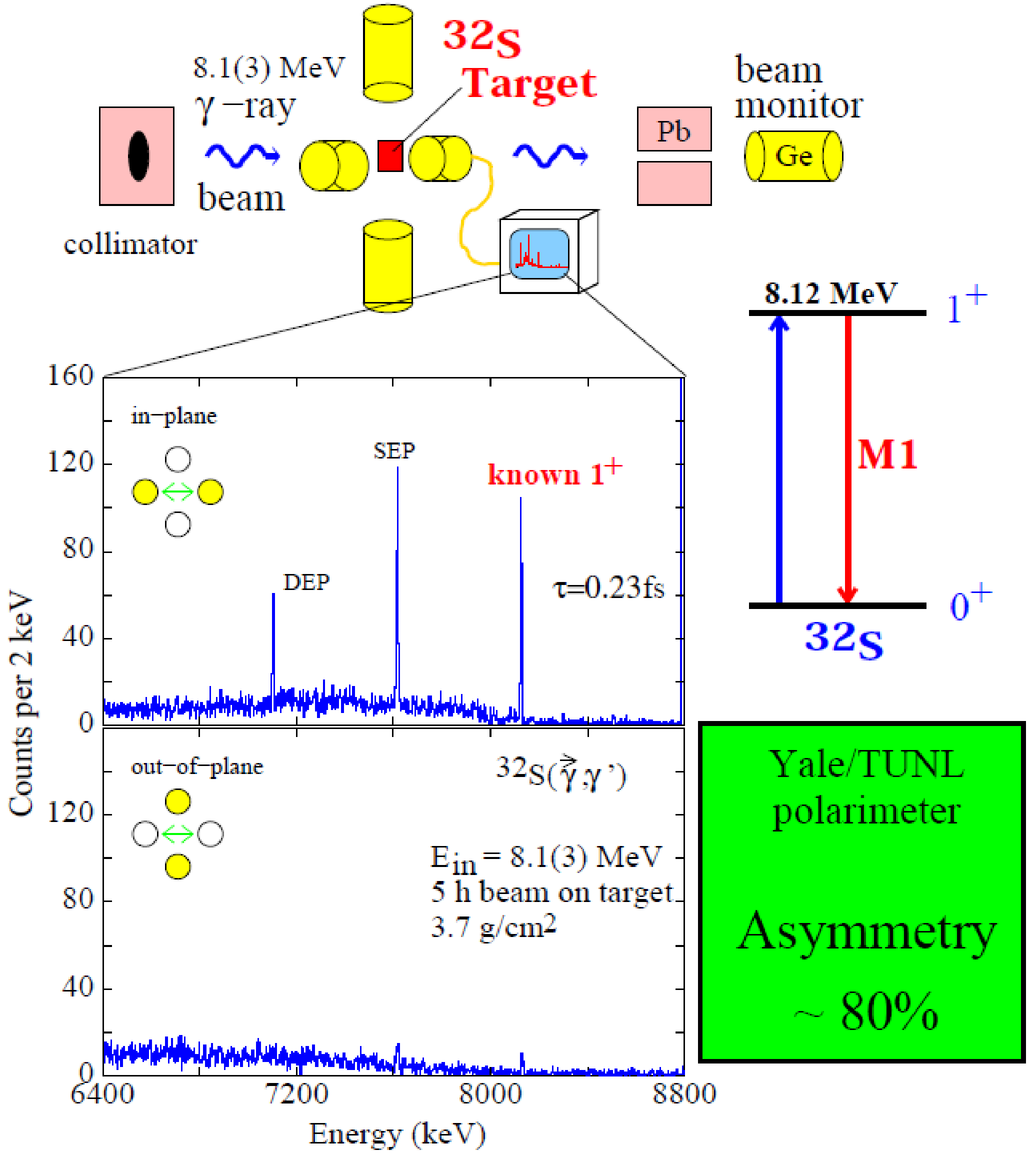


Klein-Nishina formula:

$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E_{\gamma'}}{E_\gamma}\right)^2 \cdot \left\{\frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2sin^2\theta \cdot cos^2\varphi\right\}$$

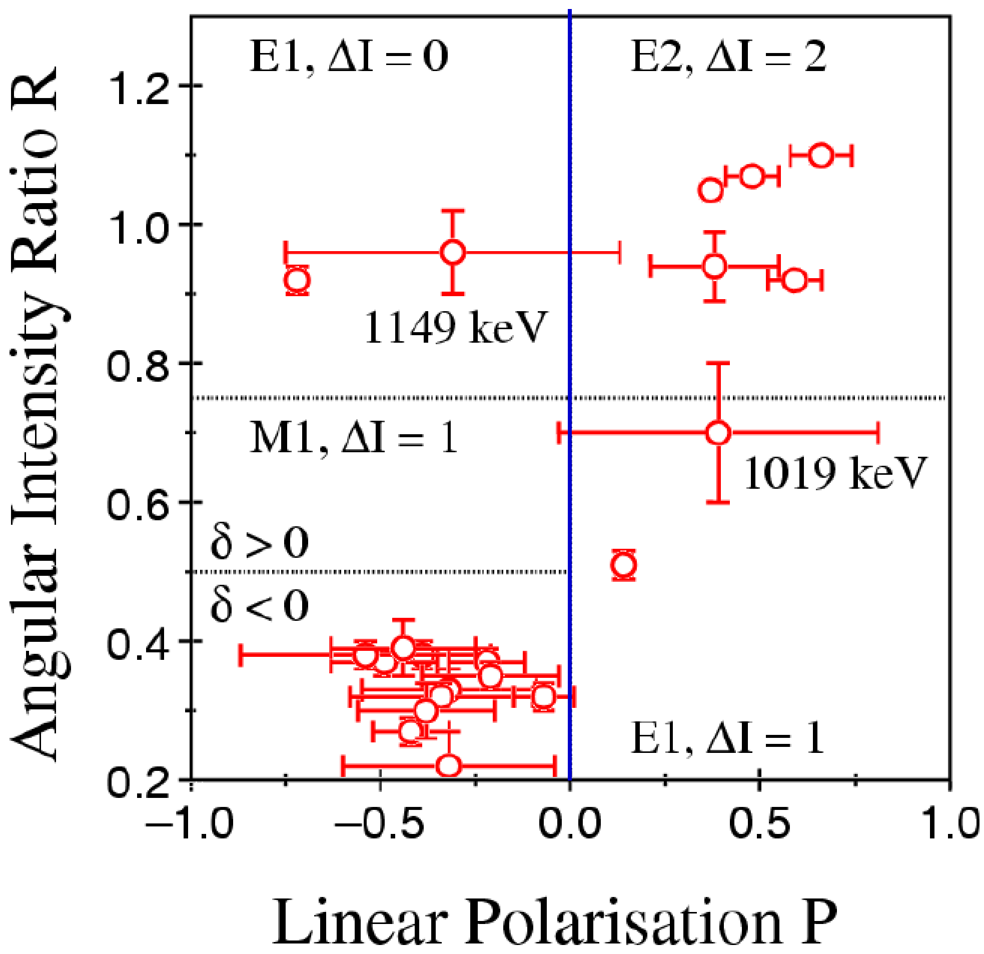
Maximum polarization at $\theta=90^0$

Proof of Principle



N. Pietralla, Nucl. Instr. Meth. A483, 556 (2002)

Linear polarization



Plot P against the angular distribution information to uniquely define the multipolarity.

Data from Eurogam

Appendix: Legendre polynomials

$$P_0(cos\theta) = 1$$

$$P_1(cos\theta) = cos\theta$$

$$P_2(cos\theta) = \frac{1}{2}(3cos^2\theta - 1)$$

$$P_3(cos\theta) = \frac{1}{2}(5cos^3\theta - 3cos\theta)$$

$$P_4(cos\theta) = \frac{1}{8}(35cos^4\theta - 30cos^2\theta + 3)$$

$$P_5(cos\theta) = \frac{1}{8}(63cos^5\theta - 70cos^3\theta + 15cos\theta)$$

$$P_6(cos\theta) = \frac{1}{16}(231cos^6\theta - 315cos^4\theta + 105cos^2\theta - 5)$$